

**Review of the book**  
**“A realistic environment in the initial teaching of geometry”**  
[In Serbian: *Реално окружење у почетној настави геометрије*]

*A realistic environment in the initial teaching of geometry.* Olivera Đokić (2017) [In Serbian: *Реално окружење у почетној настави геометрије.* Оливера Ђокић]. Belgrade, Serbia: University of Belgrade, Faculty of Education, 539 pp. ISBN 978-86-7849-246-4

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This is a review of the first part (Introduction and Theoretical Background) of the book "*A Realistic Environment in Initial Teaching of Geometry*" written by Olivera Đokić of the University of Belgrade. The second part under the title "Methodological Framework of Research" refers to concrete research in which it exposes the comparative results of mathematics teaching that are realized within the Theory of Realistic Mathematics Education [RME] and in the traditional way.

The book is written in Serbian using Cyrillic script.

The RME is rooted in Freudenthal's [3], [4], [5] interpretation of mathematics as a human activity. In Freudenthal's view, students should be given the opportunity to reinvent mathematics by organizing or mathematizing either real life situations or mathematical relationships and processes that have meaning for them. In developing this position, Freudenthal emphasized that the material that students need to mathematize should be real for them. This is the reason that the approach is called 'Realistic Mathematics Education'. Freudenthal considered mathematizing to be the key process in mathematics education for three reasons. First, mathematizing is a major activity of mathematicians. Second, the concept of mathematizing fosters applicability by familiarizing students with a mathematical approach to everyday settings. Third, mathematizing relates directly to the idea of reinvention, a process in which students formalize their informal understandings and intuitions. For Freudenthal and for researchers and curricula developers who subsequently elaborated his ideas, the goal of mathematics education should be to support students' mathematical learning as a process of guided reinvention.

The RME theory has been fertile field for research. Many doctoral dissertations about this theory have been defended over at last two decades: Van den Heuvel-Panhuizen (1996) [7], Armanto (2002) [1], Fauzan (2002) [2] and Kizito (2012) [6]. In addition, since 2006, every second year, an international conference devoted to research in this field of mathematics education is organized.

In the educational space of the former Yugoslavia, this is the first book that speaks of one of the most modern theories of mathematical education – The Theory of Realistic Mathematics Education. The book is written in an academic form requiring from the reader to be more familiar with the domain "Research in Mathematics Education". So, the book will be very useful to students of postgraduate and doctoral studies. The second part of the book consists of nine sections in two separate subdivisions. I am convinced that the first part of this book as an independent entity will significantly influence the conceptual orientation of a

large number of researches in the field of mathematics education in the geographic area of former Yugoslavia.

The introduction to the book (pp. 13-27) is written as a beautiful prologue to the Theory of Realistic Mathematical Education and as an independent part it can be seen as a very qualitative review work of this theory of mathematics education in the Serbian language space.

The first part of the book (Theoretical Foundations) consists of two divisions: 'Geometric Foundations' (pp. 31-60) and 'Didactic-Methodical Basis' (pp. 61-265). The latter division consists of six sections:

Chapter 4: The role of childhood in understanding of space;

Chapter 5: The development of spatial thinking and abilities for abstraction in a child;

Chapter 6: A theoretical background of the teaching approach of "Realistic Mathematics Education";

Chapter 7: A textbook for initial teaching of geometry;

Chapter 7: Motivation for learning;

Chapter 8: Students' thinking about the teaching of geometry and their motivation for learning;

Chapter 9: Students' ability for constructing abstractions in geometry.

A special value of this book is reflected, among other things, in Chapter 6 (pp. 79-175). This is the most extensive part in the first part of the book. It covers the following topics:

6.1. Approaches to mathematics education;

6.2 Realistic Mathematics Education;

6.2.1 Fundamental characteristic and principles of teaching and learning in RME;

6.2.2 Mathematical modeling in RME;

6.2.3 RME - based trends of research

6.2.4 Components for teaching based on the principles of the RME theory;

6.3 Realistic Geometry;

6.3.1 Geometry in context and in problem solving;

6.3.2 Construction of abstract mathematical knowledge in context;

6.4 Theories of the development of mathematical thinking;

6.4.1 Theories by van Hiele, Fischbein and Duval about the learning of geometry and geometrical thinking;

6.4.2 Methodological implications in the initial teaching of geometry;

6.5 Learning by discovering;

6.5.1 A new approach to learning by discovering;

6.5.2 Usage manipulative materials and motivation for applying manipulatives in mathematics textbooks;

6.5.3 Manipulative materials, as a cornerstone in constructing geometric knowledge.

In the first part of this book, the author emphasized the value of the positive heuristics for design proposed by RME. The author suggests that, unlike the traditional way of teaching by connecting students' current understanding of geometry to established mathematical ideas, concepts, processes, and procedures, the RME approach to teaching enables gradual accumulation of geometric knowledge towards substantial participation of students in mathematical practices of our schools.

The proof of the last statement was attempted by the author to be proved through experimental research in elementary schools in Belgrade, Serbia (pp. 272-378). The goals of the research were (p. 272):

1. In which way does the teaching of geometry to elementary school students influence their success when using techniques based on the principles of the Theory of Realistic Mathematical Education?
2. How does the teaching of geometry to elementary schools students motivate their learning when teachers use techniques based on the principles of the Theory of Real Mathematical Education.

In the Serbian language space (Bosnia and Herzegovina, Croatia, Monte Negro and Serbia) this book is a significant contribution to our science culture in mathematics education research.

## References

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