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STEM Education: Action Learning in Primary, Secondary, and Post-secondary Mathematics

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Abstract. The paper shows that mathematics as an applied tool – when used consistently at the elementary, secondary, and post-secondary educational levels – has true potential to improve the teaching of the subject matter, increase students' interest in STEM disciplines, and bridge the gap between the formal and engineering approaches to mathematics education. The adoption of the action learning techniques that have successfully been used in academia in the contexts of business management, social sciences, and teacher development is proposed as signature pedagogy for K-20 mathematics education. The appropriate use of technology and hands-on activities as an enhancement of mathematical applications to real-life projects of different degrees of complexity is discussed. Many examples are given.

Keywords: action learning, mathematical applications, interdisciplinary collaboration, teacher education, STEM, educational technology, calculus

ZDM Subject Classification (2010): **170, N70, R20**

1. Introduction

1.1. A PROPOSAL

In 2012, the President's Council of Advisors on Science and Technology (PCAST) for the United States issued a report stating that economic forecasts predict the need for an additional one million STEM (science, technology, engineering, mathematics) jobs in the next decade, but currently fewer than 40% of prospective STEM students complete a STEM degree [26]. The PCAST report proposes five overarching recommendations, one of which was to "launch a national experiment in postsecondary mathematics education to address the mathematics-preparation gap" [26] that exists between the secondary and post-secondary levels. The need for this experiment is due to a number of reasons that the

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present paper aims to address. One such reason deals with an overall non-homogeneity of teacher preparation programs – both across the nation and around the world – that sometimes pay insufficient attention to mathematical content. As a consequence, "programs that compromise on subject matter training with the result that teacher candidates develop only a limited mathematical understanding of the content covered at specific levels, have detrimental effects on PCK [pedagogical content knowledge] and consequently negative effects on instructional quality and student progress" [12]. Another reason deals with a disagreement among post-secondary faculty of how mathematics has to be taught at that level. Many students come to tertiary STEM programs with high expectations for learning – only to discover that the four letters of the word STEM are not really close to each other, and the four subject matters that the acronym comprises have little connection. Mathematics is often taught in a rather dry and unnecessarily rigorous way when "mathematicians overemphasize [the role of systems, axiomatic, and other formal aspects of mathematics] without appreciating the nature and role of meaning in students' mathematical learning" [73]. By the same token, inappropriate use of technology as an enabler of problem solving can lead to "an automatic transport phenomenon" [35] – where the outcome of a tool-based mathematical practice depends entirely on one's ability to enter data into the tool correctly – without a bit of conceptual understanding. Finally, the third reason is due to another extreme position when the "M" component of STEM is completely outsourced to those who – while doing true applied research – have little, if any, appreciation of the genesis of mathematics as an experimental science [13,10].

As a way of addressing the above inconsistences in the teaching of mathematics, the authors propose 1) the adoption of action learning techniques and 2) the incorporation of diverse technological tools across all levels of mathematics education. These two pedagogies can be combined through the use of hands-on and technology-enabled mathematics application projects. The effectiveness of these pedagogies can be measured at all educational levels in terms of increasing the motivation, while decreasing the anxiety, for learning mathematics. In particular, one goal of the action learning pedagogy at the K-12 level is to increase the percentage of students who attempt and ultimately succeed in advanced mathematics courses (Pre-calculus, Probability & Statistics, and Calculus), which has been shown to be a strong predictor for the completion of a post-secondary STEM degree [19]. At the post-secondary level, a successful outcome of the action learning pedagogy can be measured by the percentage of students attempting and completing the terminal mathematics course for their STEM track (Calculus, Differential Equations, Statistics, etc.), as well as an increase in the engagement of undergraduate research activities involving mathematics applications.

This paper is written by a STEM team of researchers and educators with different affiliations, teaching responsibilities, and research interests. At the same time, the notion of mathematics as an applied tool – used in a multitude of real-life contexts – comprises the common thread that enables the authors to join forces in thinking about the improvement of mathematics education as a whole. Indeed, the joint experience that the authors possess spans several industrial environments and all three academic educational levels: elementary, secondary, and post-secondary.

At the elementary level, this experience includes work with young children and their future teachers on interactive mini-projects. At the secondary level – work with middle and high school students and their teachers on open-ended interactive mathematics/engineering design projects, and at the post-secondary level – work with soon-to-be professionals who have to learn mathematics as an important component of their university STEM education on a real-world problem guided by mathematics and subject area advisors (see Table 1).

In that way, the ideas of the paper can be seen as a proposal in support of the concluding words from the keynote to the 1996 Conference on the Future of Mathematics Education at Research Universities, calling on the participants to "begin to see our concerns for graduate, undergraduate and K-12 education as parts of an integrated educational enterprise, in which we have to communicate and collaborate across cultural, disciplinary, and institutional borders" [11]. It also attempts to bridge the dichotomy between the recommendations to outsource post-secondary mathematics teaching to non-mathematicians [26] with the response of the American Mathematical Society to the PCAST proposal [29].

Primary	• Interactive mathematics mini-projects (games) • Single teacher supervision • Incorporate technology in a meaningful way
Secondary	• Open ended, interactive engineering design projects • Mathematics and science teachers' guidance • Incorporate modern technologies, common among STEM fields
Post-Secondary	• Individual mathematics application projects • Double guidance: mathematics and subject area advisors • Incorporate field specific technologies • Research opportunities: conferences and publications

Table 1. Proposed structure for action learning of mathematics across all academic levels

1.2. Explanation of the problem in STEM education

The Georgetown Center on Education and the Workforce reported that there will be 55 million job openings by 2020 and STEM will be one of the fastest expanding sectors of the labor force [76]. It is estimated that 95% of the STEM jobs may require a post-secondary degree [17]. To meet these challenges, colleges and universities need to provide an increasing number of graduates knowledgeable in STEM areas [60] by overcoming the current situation where a majority of students are not attracted to these areas [72]. For example, if the current trend continues, according to the National Center for Education Statistics, only 4% of the U.S. high school graduates (regardless of gender or ethnicity) will obtain an undergraduate degree in mathematics or physical sciences [20]. This lack of student interest in STEM subjects has complex origins, and can be seen to emerge at pre-college levels. One of the reasons can be found in students' mathematics learning experiences, which are often associated with little or no connection to real-life problems and methods of solving them. Without practical examples that motivate problem-solving strategies, most pre-college students struggle to retain the theoretical lessons they have learned [6]. Furthermore, studies show that pressure for standardized testing – which some educators think reduces the quality of course content [54] and blocks creativity [28,34] –

Without practical examples, most pre-college students struggle to retain theoretical lessons is placing mathematics education in an unfortunate position where students are asked to learn how to schematically do problems, and are not asked to critically think about how to solve them [76].

On the other hand, most STEM professionals seem to agree that the overall quality of STEM education and research is closely connected with the quality of college level mathematics education: over 89% of multidisciplinary STEM faculty and 93% of STEM students polled by the authors strongly support it. This suggests that the difficulty in gearing students towards STEM disciplines during their mathematics classes is a symbiosis of K-12 legacy issues (under-preparedness to deal with routine mathematics questions combined with underdeveloped creativity for deciphering real-life problems) and the widespread abstractness of teaching mathematics at the college level. Thus, while taking pure mathematics classes, many STEM students outside of mathematics lose interest in completing a STEM degree. For example, solutions to numerous industrial, business, and research problems require calculus methods and computers; but when calculus is taught from a purely mathematical standpoint (starting from the theory of limits), many STEM students have difficulty understanding the subject, and the reason for studying it becomes elusive.

1.3. Action Learning Pedagogy for Mathematics Education

Action learning is a problem solving method characterized by taking an action and reflecting on the results, and was developed by Reginald Revans in the period of 1940-1980 as an educational pedagogy for business development and problem solving [67,68,14]. Since that time, action learning has come to describe a variety of experiential learning activities [68,53,62,56], though Dilworth [25] describes the fundamental features of action learning as:

- Questioning insight is always the starting point.
- The problem must be real. The problem to be solved can be tactical or strategic, but the learning is strategic.
- Reflection is as important as action.
- Three basic questions commonly begin the action learning process in addressing a real problem.
    - First, what should be happening?
    - Second, what is stopping us from doing it?
    - Third, what can we do?
- Learning is the primary goal, even though the problem solving is real and important. Learning is facilitated, to include breaking out of well-established mind sets by having the setting, the problem, and colleagues to some degree unfamiliar.

The concepts of action learning and action research have been traditionally used in academia for teaching business management and the social sciences [48,44,57], conducting scientific research [31,22], and teacher development [61,64,63]. However, action learning – as a teaching method – has yet to be adopted as pedagogy for mathematics education. In this paper, the authors present a technology-assisted, action learning pedagogy for teaching mathematics through real-world problems – guided by STEM instructors and community professionals.

1.3.1. Teaching with applied projects

By critically analyzing the traditional teaching style with its emphasis on formal mathematics, the authors suggest that an application-based reform – directed towards the improvement of the effectiveness of K-16 mathematics education – has great potential to become the signature pedagogy of mathematics (discussed in Section 2). Though the weakness in mathematics education is more visible at the tertiary level, the starting point of the suggested reform is elementary mathematics education. The goal of the reform is a better quality and preparedness of the future STEM workforce for global competition in the 21st century. It involves mathematics application projects integrated throughout the entire K-16 curricula. At the primary level, the mini-projects may deal with solving and posing mathematical problems in context. At the secondary level interactive mathematics projects may deal with open ended engineering design problems. At the tertiary level, the projects may deal with the application of mathematics to non-mathematical components of STEM.

Mathematics application projects should be integrated throughout the entire K-16 curricula

2. Signature pedagogy of teaching mathematics through applications

In this section we use the framework of signature pedagogies to assess the theoretical merits of the action learning pedagogy in mathematics education. This section is primarily intended for researchers in educational theory. Practical implementations of the action learning pedagogy appear in Section 3.

2.1. THE NOTION OF SIGNATURE PEDAGOGY

The notion of signature pedagogy was introduced by Lee Shulman [70] in the context of professional education who argued that "to understand why professions develop as they do, study *their* nurseries, [that is], their forms of professional preparation" (p. 52, italics in the original). This notion was then explored for a variety of disciplines [36], including mathematics. According to Shulman [70], one can characterize signature pedagogies by three distinct attributes – surface, deep, and implicit – called the structures of pedagogy. The idea of the three structures of pedagogy was extended further in [3] by considering two parallel universes – the universe of teaching and the universe of learning – each comprised the three structures. This extension enabled considering students as the very recipients of their instructors' signature pedagogy. It goes without saying that teaching affects learning; by the same token, the way students learn can alter the way instructors teach – consider, for example, the effect of student course evaluations on college instructors. Due to such reciprocity, two parallel universes of teaching and learning with three, perhaps overlapping, domains – surface, deep, and implicit domains – may be considered in the spirit of Shulman [70]. Using this extended construct, one can characterize teaching and learning mathematics through projects linked to genuine real-life applications in terms of the three domains (structures). Surface level structure of learning can be illustrated by the case when mathematical model of a real-life situation selected by a student is way too approximate. For example, when finding the area of a restored wetland one can use contour integral taken along the wetland's boundary.

To understand why professions develop as they do, study their nurseries

– Lee Shulman

Alternatively, one can approximate the area through simple polygons such as triangles and parallelograms. The latter kinds of approximation if used in a project by a student belong to the surface structure of learning mathematics.

Likewise, the surface structure of learning mathematics through projects can be illustrated by the use of artificial situations with little or no connection to real-life problems or when students (by using mathematics) arrive at conclusions violating geometrical, physical, or biological meaning of the situation involved. A classic example of that kind from the elementary school curriculum is one's response to the problem of arranging twenty students to do six team presentations: after correctly dividing twenty into six one offers a contextually meaningless statement "three and remainder two students in each team" as an answer (see also a bussing problem in [18]).

A deep structure of the action learning pedagogy occurs when a problem is jointly explored by a STEM major and his/her mathematics advisor. At the secondary level, a deep structure of the pedagogy occurs when a student assimilates the critical aspects of an engineering design and generalizes the solution for novel applications. Finally, a deep structure of the action learning pedagogy emerges at the primary level when pupils ask questions and integrate answers they receive with their activities from which the questions stem.

2.2. Computer-Assisted Signature Pedagogy

In the modern technological context, a signature pedagogy can be supported by powerful digital tools. This innovation brings about the notion of a computer-assisted signature pedagogy (CASP). The surface structure of a CASP – both in terms of teaching and learning – is mostly concerned with the pedagogy of entertainment when a computer provides an easy-to-use learning environment which, while visually appealing and conveniently interactive, is not aimed at in-depth study of a subject matter but rather, enables students merely to have a *good time* in the classroom. In mathematics, at the pre-college level, teaching and learning at the surface structure level of CASP supports teachers' focus on using a drill-and-practice software program designed for a mathematics lesson. Alternatively, at the surface structure, different images (iconic or symbolic) afforded by a specific computer application can be visualized; yet a teacher does not expect students to start making connections among the images. Likewise, at the tertiary level, an instructor's goal may be to have their students practice integration through the use of a similar type program, and in doing so be dependent on an "*automatic transport phenomenon*" [35] for which one's ability to carry out flawless integration simply requires entering all of the data into a computer correctly.

Computer-assisted signature pedagogy cannot be viewed as a successful tool unless a teacher employs the deep structure of the signature pedagogy

The surface structure level of using technology in education was termed by Maddux [51] as its Type I application, seeing it markedly different from Type II application, which is supposed to manifest "new and *better* ways of teaching" (p. 38, italics in the original). More recently, Maddux and Johnson [52] warned that "the boring and mundane uses to which computers were often being applied (at the very outset of their educational applications) had set the stage for a major backlash against bringing computers into schools" (p. 2). This suggests that CASP may not be viewed as a means of successful educational innovation unless a teacher employs the deep structure of the signature pedagogy.

The CASP approach aims at gravitating away from the surface structures of teaching and learning and, instead, they integrate, whenever possible, their deep structures. This requires an active interaction among different levels of the Teaching and Learning Universes.

In the case of mathematics teacher education, a project can be rooted in the application of technology to mathematics teaching and learning. In the learning universe, one can use ready-made programs (at the surface structure level), and create new computational learning environments (at a deeper level). For example, one can use a spreadsheet to generate the natural number sequence by using only formatting features of the program: font type, size, color, etc.; that is, creating an attractive interface of the learning environment. At a deeper level, one can parametrize the level of mathematical complexity of the natural number sequence to allow for the variation of the first term, the difference between two consecutive terms, the length of the sequence, and so on.

2.3. Surface Structure of Teaching and Learning through Applied Projects

A less trivial attribute of teaching and learning through projects is that it offers the students an opportunity to own their project topic, and custom-tailor it to their individual interests – in a specific real-life problem situation. Nonetheless, at the surface structure of learning, a student, for example, may disregard the fact that the results of integration can yield negative values for area or volume. Also, students can have different dimensions in the left- and right-hand sides of their equations/inequalities. Younger students may disregard the irrelevance of large numbers in measuring certain characteristics of real life.

Furthermore, mathematical models and related machinery that a student selects for a project may be too simple and naive. For example, often in the context of statistical analysis, a student may select linear regression and, consequently, obtain confusing results. Likewise, a student can select a model for a physical or biological phenomenon that is described by a linear model (e.g., in the form of differential equation), again, leading to the results that are in contradiction with a physical or biological meaning of the phenomenon. Another student may come up with a non-linear model, yet a mathematical method selected to deal with a non-linear phenomenon may be based on linearization that, once again, leads to the results that are in contradiction with a physical sense of the situation involved. That is why a subject matter advisor is needed because a mathematics instructor may not know all the peculiarities of the real-life situation.

A project can allow a student to custom-tailor their education and diversity their pathway to a STEM career

A grade-appropriate experience at the primary level is desirable to be prepared to learn mathematics through projects at the tertiary level. For example, as described by Abramovich, Easton, and Hayes [3], within a spreadsheet-based project, the use of multiple worksheets – both linked to and independent of each other – allows for the creation of learning environments of different levels of technological sophistication. Students can progress through the worksheets of a spreadsheet like through the pages of a book.

2.4. Deep Structure of Teaching and Learning through Projects

The deep structure of signature pedagogy was defined by Shulman [70] as the best way "to impart a certain body of knowledge and know-how" (p. 55). The value of using project-based option is that it relieves the demands of understanding abstract mathematical concepts in the pursuit of using mathematics as a tool. This feature enables students' comprehension of basic mathematical concepts as valuable instruments rather than artificially selected topics to be included into undergraduate mathematics curriculum. That is why, at the deep structure of the project-based signature pedagogy, the effort of an instructor should be for a student to appreciate the applied value of mathematics and the critical role of the subject matter in improving the world around us. Furthermore, the intent should include demonstration that such an improvement is in the hands of students, should they acquire mathematical ideas.

The intent of an instructor should be for a student to appreciate the critical role that mathematics serves in improving the world around us

For the project-based approach to mathematics teaching and learning to be successful at the deep structure of the two universes, an empirical approach to the development of knowledge can be utilized. This approach was strongly emphasized by John Dewey, the most notable reformer of American education in the first part of the 20th century. Dewey [24] argued that experience is educative if one's intellectual growth is its main outcome and towards this end promoted the reflective inquiry approach to the development of knowledge. Reflection on experience occurs at the deep structure of both teaching and learning universes. New knowledge about real-life problems can be developed through a professor-guided reflection, as students are encouraged to inquire about the applied meaning of their mathematical experience. Whereas a professor determines the level of guidance, the ideas (as Pólya [65] put it) "should be born in students' minds and the teacher should act only as midwife."

2.5. Implicit Structure of Teaching and Learning through Projects

According to Shulman [70], implicit structure of signature pedagogy is comprised of "beliefs about professional attitudes, values, and dispositions" (p. 55). Can a student solve a real-life problem? Can one learn mathematics through apprenticeship? Can real-life problems be expressed mathematically? Can real-life problems be solved through mathematics? A professor's professional attitudes and beliefs regarding project-based learning of undergraduate mathematics strongly affect his or her students' learning abilities. In the case of the mathematics application project, one may believe that a critical attribute of signature pedagogy is the absence of the final exam. That is, if a professor's belief about the usefulness of projects as a teaching and learning tool does not answer the above questions in affirmative, teaching and learning through projects would not occur at the deep structure levels of the two universes.

A professor's professional attitudes and beliefs strongly affect a student's ability to learn

For example, a teacher may believe that the speed and efficiency of calculations provided by an electronic spreadsheet is the most important feature of CASP integrating this tool. In that case, the teacher's implicit structure of teaching will be oriented toward students' learning of basic skills in using Excel such as creating and saving files, navigating directories, managing different platforms, as well as improving spreadsheet programming abilities. However, if a teacher believes that the speed and efficiency of calculations have to be utilized for complex mathematical explorations, an emphasis of instruction would be on problem solving, mathematical connections, and grade-appropriate experimentation with mathematical ideas in the context of a spreadsheet.

3. Action learning of mathematics at each educational level

Action learning includes two fundamental features: an action (decision) and the reflection upon that action. As students gain technological proficiency, progressing along the STEM track, their chosen actions should gain in consequence and their reflections should increase in depth (see Table 1). The proposed manifestations of the project-based signature pedagogy for mathematics education at the post-secondary, secondary, and primary levels are detailed in Sections 3.1, 3.2, and 3.3, respectively.

3.1. Post-Secondary School

Action learning in mathematics resembles the activities of academic and industrial STEM professionals

At the undergraduate level, action learning in mathematics begins to resemble the activities of academic and industrial STEM professionals. Rather than responding to a series of formulaic questions that may have little relevance to her/his major interest, a student takes the action of choosing 1) a mathematics advisor, 2) a subject area advisor, and 3) a real-word problem from the surrounding community. Occasionally, students select a project topic and then find a suitable subject area professional; however, many students use the opportunity to network into a professional community by first approaching a professor or an industrial specialist who can propose a current problem from their field. During the project work, students are often required to utilize certain field-specific technologies that are essential for solving state of the art challenges, rather than using technology merely as a prop. After reaching a solution to the initial problem, a student – in the action learning pedagogy – reflects upon her/his work by writing a summary report that includes: a statement of the problem, the motivation for choosing the problem, a mathematical approach to reach the resolution, and finally a discussion of the results. In the conclusion section, many students reflect (as a part of the deep structure) by exploring the issues that they encountered while working on the project, discussing how well their results matched their intuition of the problem, and suggesting directions for future research.

Once a student uses mathematics in an essential way, the subject then becomes a tool for making life easier rather than harder

As in other forms of action learning, the impact of a student's action – in creating a doubly-supervised mathematics project – can be substantially more consequential than other forms of assessment, e.g., homework, exams, etc. For instance, some students decide to undertake an issue relating to their local community, using the project as form of service learning [34], for which a resolution yields a meaningful and lasting improvement for the society. Whereas, some STEM majors leverage the opportunity to begin their research careers by publishing their summary reports in an undergraduate research journal or presenting their results at a conference. Once a student has used mathematics in an essential way to solve a meaningful problem in their life, the subject of mathematics can be reframed as a tool for making life easier rather than harder.

3.1.1. Action Learning at the Post-Secondary Level

The Mathematics Umbrella Group (MUG) at the University of South Florida (USF), initiated by Arcadii Grinshpan in 1999, bridges the gap between mathematics education and applications, while inspiring STEM students to attain the mathematics skills essential for success in their respective disciplines. The MUG program is aimed at bringing creative, experiential learning into the curricula of undergraduate mathematics courses (first of all, engineering and life sciences calculus) through an optional project that can substitute for some course requirements. It forges a mutually beneficial partnership along educational lines between mathematics faculty and the non-mathematical community, which is an effective way of mathematics faculty development [34]. Students embark on individualized business/science projects with mathematics application guided by both a mathematician and a subject area specialist (see Figure 11), so that this pedagogy is not restricted by a lack of content knowledge [12]. The projects are chosen from an area outside mathematics, often closely related to either the field of a student's study or a real-life problem within the Florida business community. This ensures that mathematics content is relevant to the students' interests and aptitudes [5,37]. According to Milligan [55], the MUG program is the first organization to endorse personalized mathematics projects – advised by both a mathematics and subject area advisor – for teaching non-mathematics majoring STEM students.

Interdisciplinary projects forge mutually beneficial relationships between students, faculty, and the community

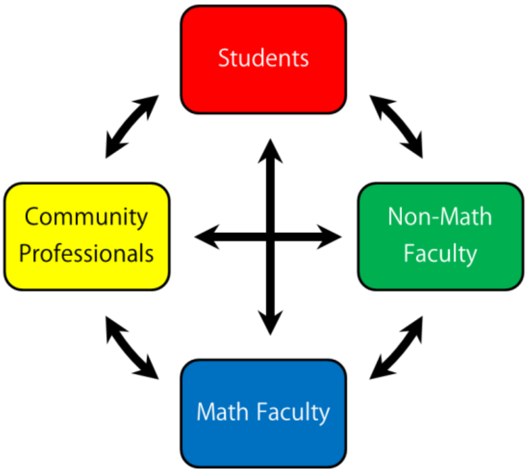
3.1.1.1. Scope of the Mathematics Umbrella Group

Over the course of its 16-year experience, MUG has fostered 2,100 interdisciplinary projects involving over three thousand of STEM students, USF faculty, and community professionals from a variety of fields [33], as depicted in Figure 1. Since its inception, the MUG program has sponsored projects in 162 calculus sections, taught by 25 calculus instructors, which contained over 7,400 USF STEM students. In addition to developing guidelines for project participants, supporting students by providing a network of community and university advisors, and maintaining an online submission and evaluation system for mathematics instructors to use, the MUG also promotes a number of undergraduate research experiences. In response to the increasing trend of the projects, the open-access Undergraduate Journal of Mathematical Modeling: One + Two (UJMM), located at http://scholarcommons.usf.edu/ujmm, was created in 2008 to showcase the best projects of each semester. The journal is free for both student authors and readers alike, and features a spectrum of STEM topics that have led to its popularity. In 2015, over 70,000 UJMM articles were downloaded, and this number continues to grow.

MUG advisors also encourage all students with high quality projects to participate in local research conferences. For example, some student projects were presented at Oktoberfest: Research Symposium and Undergraduate Research and Arts Colloquium at USF in 2012-14. In addition, the MUG program has organized student poster conferences (STEP Up for Applied Calculus: Undergraduate Research) and collaborated to host workshops for the Florida mathematics faculty, both of which were held at USF on September, 2012 and April, 2014.

3.1.1.2. Effectiveness of MUG's Action Learning Activities

The success of the project-based method of education can be evidenced through: (a) calculus passing rate for students who have completed projects is over 20% higher compared to those who had chosen not to do one, (b) a jump from 30% to 58% passing rate for African American students in the project-based sections, (c) an increase in number of students taking and passing follow-up mathematics and engineering courses, (d) about 17% higher graduation rate for engineering "project option students" as compared to historical records, and (e) an increase in undergraduate research interest in all STEM areas [39,40].



- Significantly increased motivation and STEM retention
- Real-life problems in the field of interest
- Better understanding and retention of calculus
- Connecting with the professional community
- Opportunities for research

Figure 1. Applied mathematics projects connect students with academic and industrial STEM professionals

3.1.2. Introducing Projects into the Classroom

There are a variety of ways to incorporate applied projects into the curriculum of any mathematics course, providing a means to scale the pedagogy for any college or university environment. For instance, the projects can either be compulsory, offered as an option to a final exam, or for extra credit. Administratively, project sections can pool their network of advisors and support each other through a larger organization such as MUG at USF, or each mathematics instructor can operate autonomously.

At the beginning of each semester, calculus students should be provided instructions for the applied mathematics project that contain guidelines outlining how to choose their advisors, pick a topic, format the final report, and submit their project online. Additionally, the guidelines contain links to other external resources, such as the UJMM – which showcases quality examples of projects from former students – technical writing tips, technology tutorials, and a timeline for completion of the project.

Generally instructors offer a few extra office hours at the end of the semester to accommodate the increase of project related questions ahead of the deadline. Most instructors set a deadline to submit a project around their regularly scheduled final exam time, but considerations should be made to allow a reasonable amount of time for the subject area advisors to leave their reviews of the projects.

3.1.2.1. Recommended mathematics course level

The MUG program originally offered the applied mathematics project opportunity as a part of the calculus sequence (including differential equations) in order to strengthen the relationships between the USF mathematical community and its surrounding businesses, bridging the gap between the theory and practice of mathematics. In particular, the program was initially offered to the Calculus II and III evening sections, which contain a majority of working students. On average, upper level calculus students have shown to be sufficiently responsible and mathematically adequate to contribute to the resolution of real world problems in a reasonable way. However, other courses, such as probability, statistics, and linear algebra, provide an ideal threshold for engagement in doubly-supervised interdisciplinary projects for the non-calculus tracks.

The action learning pedagogy has been implemented at USF at the Calculus I level, but on a relatively smaller scale than the doubly-supervised projects sponsored by the MUG program at the Calculus II & III levels. The Calculus I activities showed an increase in motivation and mathematical confidence among the students, which is consistent with the findings of other problem based-learning pedagogies [66].

The student demographics for pre-Calculus, and the preceding mathematics courses, are dominated by non-STEM majoring students, which extends beyond the focus of this current proposal. However, we advise the use of other successful active learning pedagogies for these levels, such as those reviewed by Prince in [66] and the PCAST in [26].

Courses designed to contextualize mathematics for a specific field have also been shown to effectively motivate students within that field. For instance, the Wright State Model for Engineering Mathematics Education has led to an increase of motivation [47] and retention [46] for engineering students.

3.1.2.2. Project-option classes

In a project-option class, the standard knowledge assessment is administered throughout the semester where in-class tests cover the most basic concepts. As a follow-up, a student either does a project – allowing them to participate in doubly-supervised interdisciplinary (mathematics application) activities – or takes a comprehensive final exam. The project option has been offered in 148 calculus sections at USF and continues to be offered in the Engineering and Life Sciences calculus classes.

An instructor’s beliefs about action learning (an implicit structure) can greatly influence how many students will choose to partake in the project option. For instance, an instructor can dissuade their class from the option by insinuating that the project will require a substantially greater amount of work or be graded stricter than a final. Along this line, unless the project guidelines are covered in class, students may be unclear about the expectations of the project and choose the more familiar final – even though it is more likely that they will score higher by submitting a project.

3.1.2.3. Project-only classes

At USF, some evening calculus classes – which are predominately filled by working students – are designated as project-only sections, where students are encouraged to seek topics for projects within their own work environments. In project-only sections, most of the subject area advisors are industrial experts – external to USF (see Figure 4) – that contribute in a one-off fashion. However, since the local STEM industrial community is considerably larger than the academic community, the limited participation of external professions is sustainable (see Figure 3).

Project-only classes have several advantages over project-option sections: 1) most students enrolled in project-only sections start working on their projects at the beginning of the semester, whereas students in project-option classes typically wait for their test results before electing to initiate a project, 2) students in project-only sections seek out a subject area advisor earlier in the semester, which allows them to find a replacement advisor in case their first choice does not accept the responsibility. Many potential subject area advisors are more willing to help out early in the semester, when the class loads are lighter.

As of the spring of 2013, MUG has sponsored 14 project-only sections at USF. A disproportionately greater number of projects that are selected for publication in the UJMM originate in project-only sections.

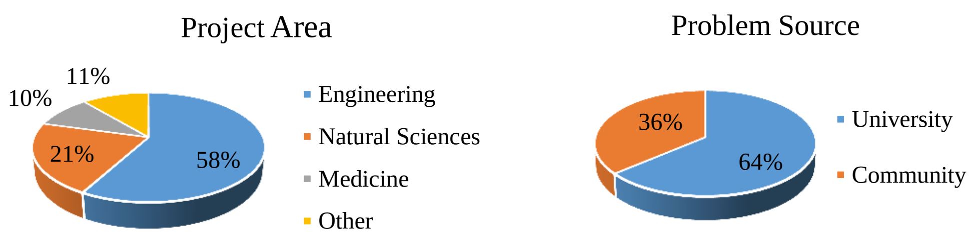


Figure 2. Classification of projects by subject area (left), and whether the project problem originated from within the university or the surrounding community (right).

3.1.2.4. Advanced placement, honors, and summer school

Several of the MUG projects were conducted by high school students supervised by the USF faculty in the USF summer school for gifted high school students, initiated by Manoug Manougian in 1979. High achieving secondary school students often seek opportunities to go beyond the standard curriculum, and the open-ended nature of a project can provide them with the environment to excel beyond grades alone.

3.1.2.5. Pure mathematics track

Teaching mathematics with doubly supervised applied calculus projects was originally developed to help non-mathematics majoring STEM students connect what they learn in their mathematics courses with their majors of choice. However, applied projects promote a deeper level of understanding, and can be useful to undergraduate mathematics majors as well. Indeed most of mathematics – including calculus – was developed to solve real world problems. Incorporating projects into pure mathematics classes gives students an opportunity to explore a topic more deeply, or connect a general core subject to a specialized mathematical field.

3.1.3. Project advisors

Every semester dozens of non-academic professionals and USF faculty volunteer to help as advisors. Each project is guided by a subject area advisor and mathematics advisor (usually the calculus instructor), while extra advisors are also allowed.

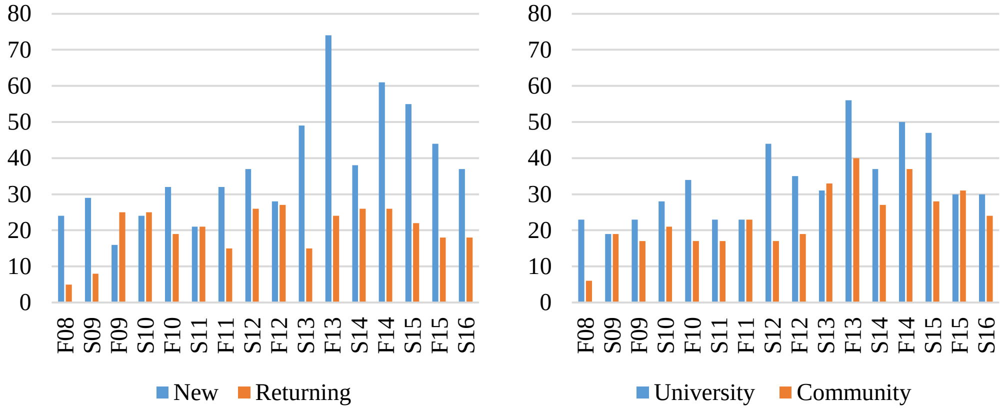


Figure 3. The total number of advisors that guided a calculus student in an applied mathematics project during the Fall 2008 to Spring 2016 semesters at USF. The charts show the number of first-time advisors compared to the number of advisors who have participated in the MUG program during a previous semester (left), and the number of USF advisors versus the advisors from the surrounding community (right).

3.1.3.1. Mathematics Advisors

Each application-based project is guided by at least one mathematics advisor that provides advice on mathematics tools and technologies. Some mathematics advisors who are experienced in supervising interdisciplinary problems are comfortable overseeing multiple projects, suggested by a variety of subject area advisors. Accumulated experience from the MUG program has shown that most projects can be conducted under the guidance of a single mathematics advisor (the course instructor), but some STEM projects involve a number of specialized topics that may necessitate the involvement of an additional mathematics advisor or the course instructor may be overloaded in her/his teaching responsibilities. Supplementary mathematics advisors can include other mathematics faculty, postdocs, and graduate assistants.

The mathematics instructor – who is usually a mathematics advisor – brings the mathematical side of the project into alignment with the course curriculum, assists in resolving unconventional cases or problems, and assigns the final project grade (taking into account the evaluations of all the advisors). In assigning the final course grade, each mathematics instructor has full autonomy of the grading system of their class. However, the influence of the project grade should be significant.

3.1.3.2. Subject Area Advisors

Every semester dozens of non-academic professionals and USF faculty volunteer to help as advisors. As of the Spring 2016 semester, there have been 728 community professionals and 335 USF subject area advisors. The subject area may include any STEM-related discipline such as engineering, natural sciences, medicine, or a related field, e.g., business, economics, finance, etc. A subject area advisor may be a faculty member, postdoc, PhD student, or a community professional in the project field (see Figure 4).

Students electing the project-option can use the MUG network to find an expert in their chosen subject area, willing to advise their project. With the subject area advisor's guidance, students decide upon a clear project topic and a plan for achieving their goal. It is expected that the project topic stems from current or planned workforce (company, or government) or scientific activities that require a mathematics application. Subject area advisors closely work with their students until the completion of the project, at which time they have an opportunity to review, evaluate and comment on the final results. These comments are especially valuable to the course instructor due to the broad range of the specialized STEM fields covered within the action learning pedagogy.

MUG records show that while many subject area advisors narrow their focus to advising a single student per semester, some experienced faculty members – who have served as lab heads within their discipline – are comfortable supervising several projects simultaneously (possibly for multiple mathematics courses). In contrast, many STEM problems are interdisciplinary, in which case, a student may seek the assistance of additional subject area advisors.

Figure 3 shows that each semester there is a large number of new advisors who choose to supervise a student in the action learning mathematics pedagogy, and that typically over half the participating advisors each semester are affiliated with USF. However, Figure 4 indicates that over two thirds of all the subject area advisors belong to the business and government communities surrounding the university. This means that the majority of the advisors each semester belong to the academic community and participate repeatedly, whilst the remainder of the advisors are external to USF and contribute exclusively to a single project. The large number of external advisors is consistent with the fact that many students are employed while they are enrolled at the university, and working students are encouraged to seek a problem from within their workplace, suggested by their employer. According to the Georgetown University Center on Education and the Workforce, a consistent 70-80% of college students are active in the labor market [16,30], providing an inexhaustible source for project topics and potential advisors.

3.1.3.3. Graduate student participation

STEM graduate students are encouraged to participate as project advisors and journal content editors and reviewers. Undergraduate students may feel more comfortable approaching a graduate student for assistance than a faculty member, or even their instructor. Graduate student participation helps to distribute the workload of generating topic for and supervising hundreds of personalized projects.

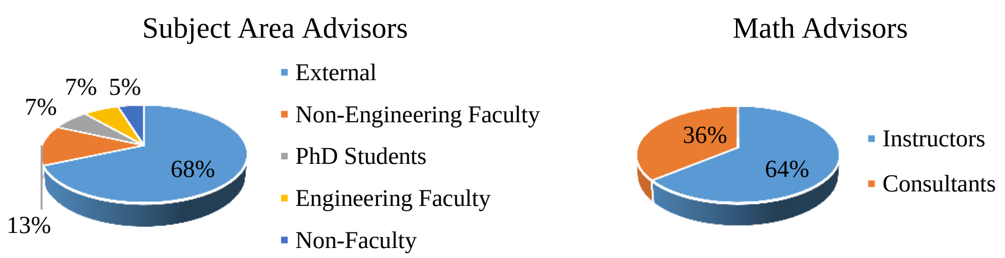


Figure 4. Classification of project advisors by their USF affiliation. External subject area advisors are not directly affiliated with USF.

Many graduate students voluntarily agree to supervise the STEM projects, but institutional support would have a greater impact. USF implements a unique system of vertical integration whereby faculty members mentor a few graduate research assistants (RAs) and teaching assistants (TAs) to become project leaders who supervise multiple projects and assist their fellow graduate students in their leadership roles. Besides helping their research by expanding their horizons, the RAs and TAs obtain unique mentoring and editing training that are not available through standard assistantships. Student advisors (RAs, TAs) are guided by the faculty to develop interesting project topics based on their own research in a way that is accessible to STEM undergraduates. Near the end of the semester, the graduate advisors hold extra office hours specifically designated for helping project students.

3.1.4. Role of technology in projects

3.1.4.1. Technology used by students

In general, when considering a project topic, many students begin by either searching keywords related to their interests online or reading some of the UJMM publications. While preparing their solutions, they use data processing suites and graphing utilities, such as Microsoft Excel©, SPSS, SAS, or Wolfram Alpha©, and – while preparing their final reports – students often use general office and graphics software packages, e.g. Microsoft Word© or Adobe Photoshop©. After the students have uploaded their projects to the MUG database, they are prompted to take an anonymous survey, leave their testimonial, and give their suggestions about how to improve the MUG program.

In the context of an individual project, the technologies required to properly tackle a STEM problem, such as computers, software, cameras, audio equipment, robots, reagents, machines, etc., are often field-specific and designed to perform a particular function. For instance, the chemical engineering major who is interested in building a low-cost continuous stir reactor may have little interest in the population growth of the Florida Scrub-Jay. Both subject area and mathematics advisors can offer their field experiences to make suggestions about the appropriate technologies required to approach the problems.

3.1.4.2. Technology used by advisors

Advisors often use field specific technologies to get their students started on the projects, and to check the correctness of their students' final solutions. Throughout the process, advisors use email and video conferencing to keep updated on their students' progress. Finally, when a student uploads their final project to the MUG database, their advisors have the opportunity to review the project and leave an evaluation online for the course instructor. The course instructor can also choose to run anti-plagiarism software before assigning the final grade, according to their own rubrics.

Please evaluate the project by assigning a rating of "Excellent", "Very Good", "Good", "Poor", "Very Poor", or "Don't Know" to each of the following six questions:

(1) How independently did the student work?
(2) How clearly did the student provide a context for the work?
(3) How clearly did the student write out and define the terms in the equations?
(4) How clearly did the student describe the methods used to analyze the problem?
(5) What do you think is the quality of the student's mathematical analysis?

(6) Overall, how well does the student understand what she/he has done? Do you have any further comments?

Table 2. Example of the evaluation questions to be answered by the project advisors, which the course instructor takes into consideration while assigning the final project grade.

3.1.4.3. Technology used by administrators

To fully implement all the research opportunities that the action learning pedagogy affords, a program, such as MUG, must rely on technology to coordinate the network of students and advisors. MUG uses the database of projects, project metadata, advisor evaluations, and student surveys to

- increase the retention of the students' mathematical knowledge,
- strengthen their confidence and motivations towards mathematics applications,
- assess the effectiveness of these goals,
- target areas of the program for improvement, and
- disseminate the results.

In particular, articles in the open access UJMM have been downloaded in almost all countries of the world.

3.1.5. Examples of post-secondary applied-mathematics projects

3.1.5.1. Volume of Lake Behnke

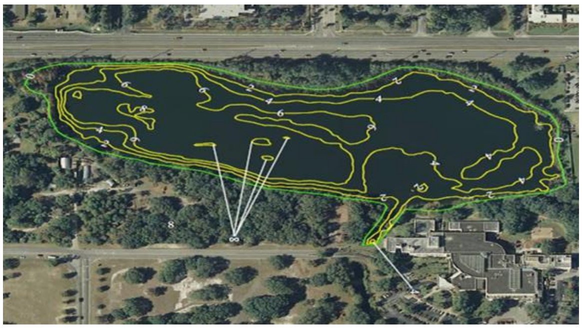


Figure 5. Bathymetric map of Lake Behnke [23]

For her life sciences calculus project, Kaitlin Deutsch [23] contacted the director of USF's botanical gardens to obtain a current bathymetric map of Lake Behnke (see Figure 5), USF's main storm water drainage basin. Using contour integration, she found that the lake has changed from the linear area-to-depth relationship – noted by the DPRM storm water management study conducted in 1998 – to a quadratic area-to-depth relationship. This suggests that the topography of the lake has changed extensively over the past 14 years.

Deutsch also presented her poster at the 2012 STEP UP for Applied Calculus poster conference. She was named a 2014 Goldwater Scholar.

3.1.5.2. Sinkhole Repair

While taking engineering calculus, Charles Griffith [32] was working for a sinkhole repair company which drills holes (some vertical, some askew) around the perimeter of a house, down to the bedrock, and fills them in with concrete. These repairs are common in Florida since it sits on a carbonate platform making the region highly susceptible to sinkholes.

The profitability of a sinkhole repair company is contingent on mixing the proper amount of concrete for each job (concrete should be poured continuously as it hardens if mixed and not used).

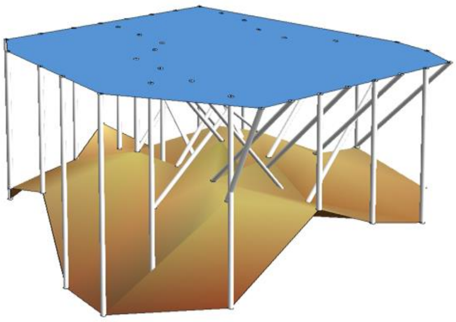


Figure 6. Model of the holes drilled to stabilize a house atop a sinkhole [32]. The surface is blue and the bedrock is brown.

By using Riemann sums to approximate the modeled area between the surface of the ground and the bedrock, Griffith estimated the total amount of concrete needed for the repair pictured in Figure 6 to within 8% of what was actually needed.

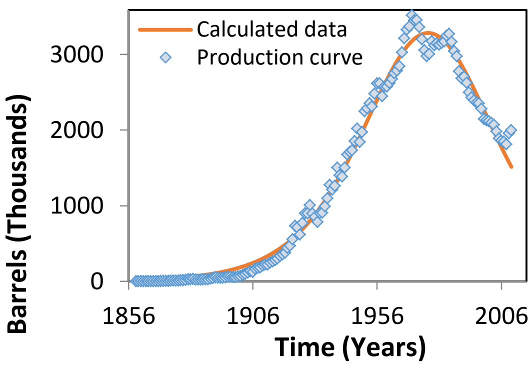
In the problem of section 3.1.5.1, the measurements for the boundary of the lake were acquired directly from aerial photography. However in the problem of section 3.1.5.2, the geometry of the sinkhole was unknown, so its boundary was estimated from the depth of the bedrock below the house.

3.1.5.3. Peak Oil

Trang Luong [50], while taking a life sciences calculus class, estimated the remaining level of U.S. oil reserves. She modeled the crude oil production from 1859 to 2010 using the records published by the U.S. Energy Information Administration (Figure 7, right). She modeled the U.S. oil production $y(t)$ in terms of t years as:

$$y(t) = \frac{A\, e^{-b(t-c)}}{(1 + a\, e^{-b\,(t-c)})^2},$$

and used least squares regression to fit the unknown parameters A, a, b, c to her data. Next, she computed the "peak oil" by finding the derivative of the modeled crude oil production over time. By integrating the model from the present to the projected end of the oil production, she was able to estimate the remaining U.S. oil reserves.



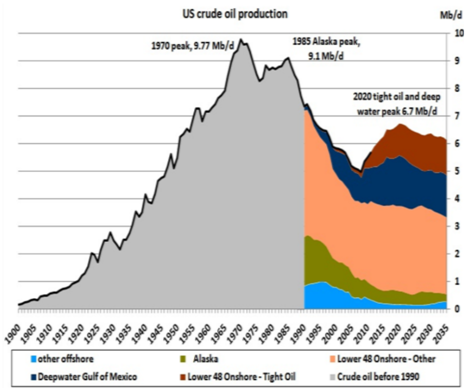


Figure 7. Regression of the US crude oil production during 1856 to 2006 (left) used in [50] to calculate the peak oil production levels from the data provided by the [77] (right).

Luong won a cash prize for her poster at the STEP UP for Applied Calculus: Undergraduate Student Poster Conference. Subsequently she received a DAAD (German Academic Exchange Service) RISE scholarship for an internship in Germany.

3.1.5.4. Complexity of Genomic Sequences

With the advent of next generation sequencing, the field of bioinformatics has grown into an industry. For his life sciences project, Brandon Toun [74] discussed the topological entropy and compressibility of the mitochondrial DNA sequences for nine organisms (see Figure 8). As part of his study, Brandon compressed DNA sequences using the Deflater algorithm implemented in the Java standard library, and compared the size of the plaintext sequences to the resulting compressed sequences. He found that the *Laminaria digitata* (a large brown algae) mitochondrial genome had the highest rate of compression, whereas the *Elaeis guineensis* (an oil palm) mitochondrial genome had the lowest compression rate. He also noted that the number of mutations to a genome can be quickly estimated by comparing the compression rate of the mutated genome to a compressed normal (reference) genome.

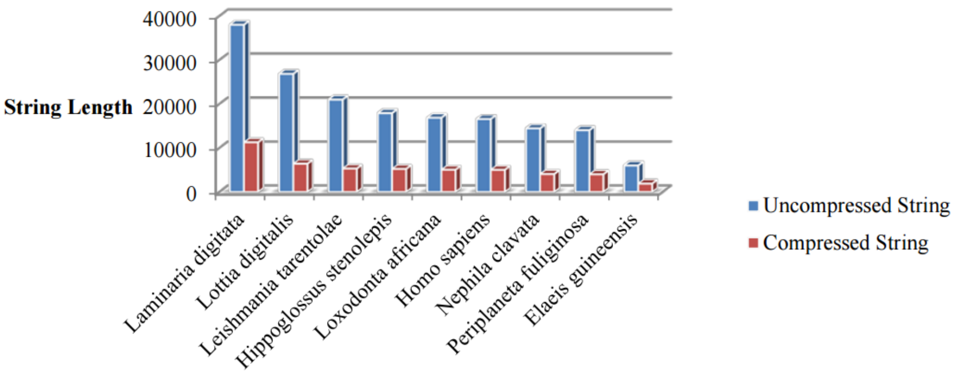


Figure 8. Bar graph comparing the original mitochondrial sequence lengths of nine organisms to their compressed lengths [74].

3.1.5.5. Pallet Physics

Each night at Publix, delivery trucks arrive and need to be unloaded. Occasionally the pallet jacks break and the employees have to unload the trucks by hand. Lauren Woodbridge [80], a high school student taking engineering calculus, explored whether it was safe to unload a pallet from a supply truck by sliding it down a metal ramp (see Figure 9). First she calculated the critical angle for which the pallet would overcome the friction of the wood on the metal and begin to slide. After calculating the pallet’s acceleration, velocity, and displacement, she concluded that it would not be safe to unload the truck in this manner.

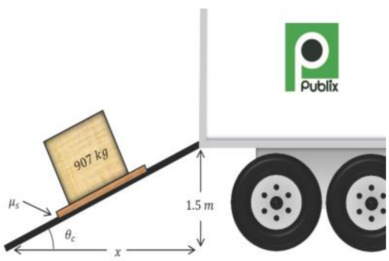


Figure 9. Unloading a Publix supply truck by sliding a pallet down a ramp [80].

This problem involves a combination of physical considerations, such as the height and length of the ramp, the material composition of the ramp and pallet, and the weight and shape of the cargo. Its solution concerns Newton's second law of motion, frictional forces, and plane geometry.

3.1.5.6. Area of a Baseball Field

Motivated by his interest in baseball, Jacob Courchaine [21] wanted to identify the costs of building a baseball field to Major League Baseball's specifications. In particular, he considered the amount of clay need to cover the catcher's box, pitcher's mound, and infield, and the amount of fertilizer necessary to grow the grass for the infield diamond and outfield.

Most of a baseball field's design follows basic geometric patterns (circles and squares), however the back wall of the outfield is an arc of an ellipse, i.e., center field is farther from home plate than the left and right outfields. Courchaine divided his model into smaller parts and used the inverse trigonometric integration techniques that he learned in engineering calculus to find the area of each of the subregions (see Figure 10).

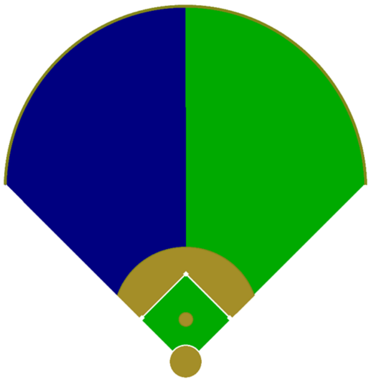


Figure 10. A baseball field with half the outfield highlighted in blue [21].

3.1.6. Student Reflections of the Project Pedagogy

Since the Spring 2013 semester, all students that upload a project to the MUG database are given the opportunity to participate in an anonymous survey. As of the Fall 2016 semester, 446 out of 600 students who uploaded a project completed the survey and the results are reported below in Table 3.

	Question	Agree	Disagree
1.	The quality of STEM education and research is closely connected with the quality of collegiate mathematics education.	418 (93.7%)	28 (6.3%)
2.	The calculus sequence is a significant part of the collegiate mathematics education for STEM majors.	406 (91.0%)	40 (9.0%)
3.	When calculus is taught as a pure mathematical content, most STEM students understand the subject and realize why they need to study it.	237 (53.1%)	209 (46.9%)

Table 3. Results of the survey given to students after they complete their interdisciplinary mathematics application project.

After answering the three questions shown in Table 2, the survey participants were given the chance to provide a free response testimonial of their project experience, and suggestions of how to improve the program. A few of the testimonials appear below:

Connection of mathematics with other STEM disciplines:

> "The combining of different fields is at the core of my project subject. NMR research combines multiple fields in order to provide a powerful tool in spectroscopy. Working on this project helped highlight the importance of interdisciplinary cooperation among the fields of mathematics, chemistry, physics, and many more."

> "I had a pleasant experience working with two advisors; subject area advisor (physics) and the mathematics advisor (calculus). The physics advisor assisted me with finding a topic and gave me brief explanations to multiple approaches for the suggested problem. Whereas, the calculus advisor helped with suggesting to look at a published paper that was written in calculus about the same general topic to get a better understanding about the type of calculus that should be used for my project. Their assistant was very useful."

> "I think it was a great experience. I felt more connected to what I would like to do as a career. Fields like engineering can get pretty theoretical in classes."

Students learn the technologies associated with their discipline:

> "Very informative, I learned how to use new computer programs that have been useful in all of my classes, and for the first time I have gotten to see how important mathematics is for the life sciences."

> "I really liked having this project to do. It made me realize how what I had learned over the semester was actually used rather than a bunch of numbers and letters. My tutor also explained to me how excel was doing partial derivatives and matrices and it helped the information click. Instead of it being an abstract concept it became concrete and applied. I also learned more about algae than I ever thought I would know. It was also kind of fitting because I studied this particular algae back in high school at an internship."

Education is what remains beyond the classroom:

> "I enjoyed the opportunity to take on the challenge of this project and found that there was so much to learn from this experience. I have truly grasped the concept found in the project and I am looking forward to applying it when I graduate."

> "I feel that applying the formulas covered in my course to a real world example we beneficial, and help me to understand how the math works. in addition i did research on topics not yet cover and i feel this was helpful to connect concepts and strengthen overall understanding. A final exam would have been just cram and forget. This will likely stay with me longer."

Building a connection with the professional communities:

> "Learning to relate with and to develop connections in the staff is perhaps one of the most valuable outcomes of this experience"

> "I took Calculus because I want to improve my ability to do STEM research - not because of a degree requirement. So, this assignment was perfect for me as I could actually practice what I was learning under the mentorship of experts."

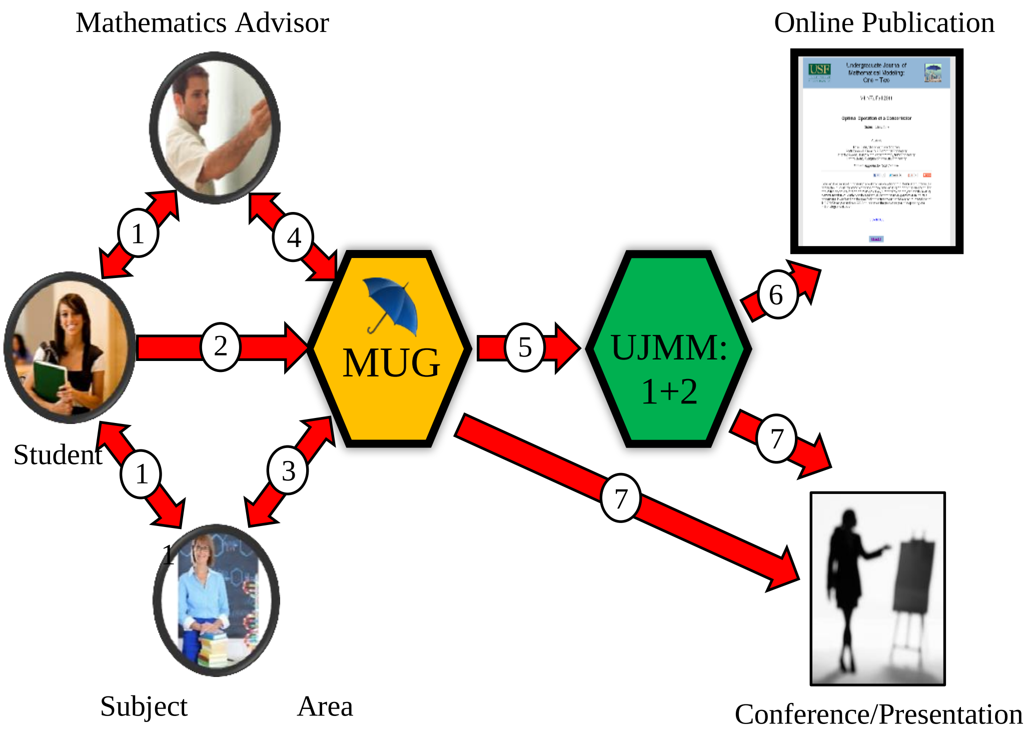
Increasing motivation to study mathematics:

> "To tell you the truth I never really felt like a numbers guy and never even tried of applying what I've learned in class to the real world and this project has shown me the light! I'm actually pretty excited to see what else I can apply to the real-world, and even though it is just theory, It's definitely pushed me past my comfort zone and really allowed me to see that I too, can come up with my own models (it doesn't have to be someone with an extremely high IQ.)"

> "Knowing how the math we learn can be applied is just as important as becoming familiar with the process of manipulating it. Context allows the student to find his or her own bearings and realize the importance of their own work; a significant motivator."

> "Many students attending these classes often think they are pointless and in turn do not pay full attention to the mathematics being presented during the course. Having to work side by side with a graduate student and seeing the actual applications of the math being thought in class was a truly beneficial experience."

3.1.7. Structure of Undergraduate Research Activities Inspired by MUG



1 Students collaborate with at *least* one mathematics and one subject area advisor to develop an interdisciplinary applied calculus project. A student's calculus instructor usually serves as their mathematics advisor.

2 Students submit their projects online to the Mathematics Umbrella Group (MUG) which authenticates their advisors' identity. Registered advisors are granted access to their students' projects.

3 Subject area advisors review their advised projects and submit evaluations to MUG.

4 MUG securely sends the subject area advisors' evaluations to the corresponding mathematics advisors who assign grades to the projects. Mathematics advisors may then choose to recommend certain projects for publication.

5 Based on the evaluations of the mathematics and subject area advisors, MUG recommends the highest rated projects for publication in the open access online Undergraduate Journal or Mathematical Modeling: One + Two (UJMM).

6 Out of the projects recommended by MUG, the UJMM selects the most interesting projects from each semester for publication. Graduate students guide the chosen authors through any necessary revisions before the articles are posted online.

7 MUG invites their recommended students to participate in a conference and helps them to prepare their presentations.

Figure 11. Structure of undergraduate research activities inspired by MUG

3.2. SECONDARY SCHOOL

The 21st century standards for technological literacy [42] describe engineering design as a purposeful, often collaborative, activity with an explicit goal, shaped by specifications and constraints, leading to multiple solutions through systematic reasoning and iterative process.

At the middle and high school levels, the mathematics and science teacher can coordinate to provide interdisciplinary engineering design experiences. Rather than starting with a mathematical tool, the teacher first poses a hands-on challenge that lets their students take an action that compels them to apply creativity and imagination to design the desired product, and have fun doing so. When students begin their designs the mathematics content is often incidental – requiring them to make a few calculations that may not be essential in the initial part of the design stage – and many may wonder if there is any connection at all to their understanding of mathematics content. Research suggests that, in a broad sense, engineering skills represent a combination of mathematical abilities and interest in technical problems [75,78]. Therefore, it is important to demonstrate that mathematical and scientific knowledge are *integral* to the engineering design process and, at the very least, may be used to make the design process less labor intensive.

Engineering design activities inspire creative thinking in a fun way, while highlighting the interdependence of the STEM fields

3.2.1. Action Learning Pedagogy at the Secondary Level

At secondary schools, many students within a grade level have the same set of teachers, yet there is often little collaboration between the subjects. Many STEM topics, however, span a broad range of fields, and engineering design projects provide an opportunity to bridge this divide while providing the students with a fun experience.

The mathematics and science teacher at the school can collaborate to choose a project that highlights an appropriate mathematical and scientific challenge, and present the problem as a hands-on activity in one of their two classes. After the students have attempted to solve the problem and recorded the results of their efforts, the science teacher can discuss the physical factors at play in the design. During the following mathematics class, the mathematics teacher can discuss how to interpret the data that their class collected, and help them to model the design problems in abstract (mathematical) terms. Combining what the class learns from their mathematics and science teachers, the students should be guided to develop an optimal solution to the design problem and verify the result empirically. The project should be concluded with a one to two page description of the problem, the different designs that the students attempted, and summary of what they learned from each design.

The engineering design challenge can either be used once per grade level, several times throughout the year, or offered as an afterschool activity – depending on the degree of collaboration between the teachers. The design project should be discussed in both classes, over the span of several days, to give the students enough time to consider alternative designs, but the overall amount of class time devoted to the design project should be restricted in order to satisfactorily cover the regular course curriculum.

3.2.2. Examples of secondary school engineering design projects

Below we provide some examples of engineering design projects and projects that reflect the integration of mathematics (and science) into engineering design.

3.2.2.1. Cardboard Raft

Most readers are probably familiar with the activity in which participants are given identical materials and are challenged to construct a raft that will hold the most weight without sinking. On the first day of class, students in the course were given a piece of cardboard (9.7×13 cm) and a larger piece of aluminum foil and were told to make a raft that would hold the most pennies before sinking (Figure 12). A tub of water was placed at the front of the room and each student came forward one-at-a-time to add pennies to their raft until it sank. The number of pennies that each raft held was recorded and a winner was declared.

Figure 12. Seeing how many pennies the raft can float.

At this point, the instructor gave a brief lecture on Archimedes principle – and students quickly realized that this scientific knowledge would have been useful in their design process. Archimedes principle states that the buoyant force on a body wholly or partially submersed in a fluid is equal to the weight of fluid displaced by the body. When equated to the force due to gravity on an object floating in water, the result is $M = \rho V$ where M is the mass of the floating object, ρ is the density of water and V is the volume of water displaced by the floating object. Therefore, Archimedes principle allowed the students to translate the problem from designing a raft that would hold the most weight to designing a raft that could displace the highest volume.

The rafts that the students designed were, owing to the constraints of the provided materials, generally flat in shape with folded up edges. The students were asked to construct a formula for the volume of an open-topped box constructed from a flat sheet of dimensions AXB with variable X representing the length of the folded up edge (See Figure 13). Next, the class prepared a spreadsheet in which the volume $V = (A - 2X)(B - 2X)(X)$ of the raft was computed as a function of the edge length X, and discovered that there was a particular value of X that maximized the volume (Figure 14). The density of water and the mass of a penny were then used to compute how many pennies this

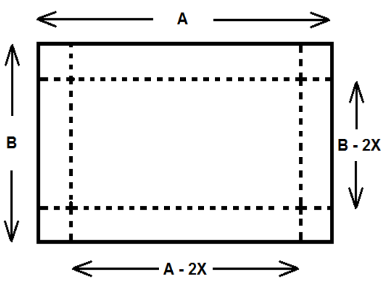


Figure 13. Constructing an open topped box (raft) from a flat sheet

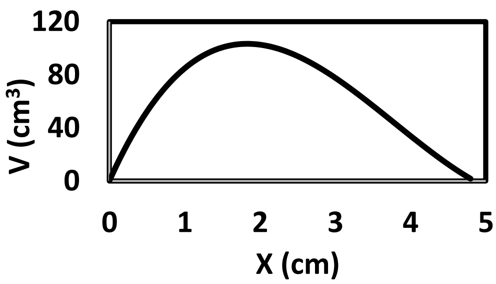


Figure 14. Volume of the raft as a function of edge length X

optimum raft could theoretically hold before sinking. This number was actually approached by the raft of one student and it was found that the dimensions of her raft were very close to the optimal design computed using the spreadsheet. As an aside, it is noted that these students had taken calculus and asked to be shown how to formulate the optimization as a calculus problem as an alternative to the brute force spreadsheet solution.

The students' initial raft designs were driven almost entirely by intuition. By the end of the exercise they saw, in this instance, how scientific knowledge could be used to more easily understand the problem and how mathematics could be used to more efficiently arrive at a solution.

3.2.2.2. Musical instruments

In another engineering-project, students were given several identical goblets, a pitcher of water, a measuring cup, a chart of frequencies for a number of musical notes, and a smartphone with an app that would display the frequency of a played sound. They were challenged to fill the needed number of glasses with water of various levels so as to be able to play "Mary Had a Little Lamb" by rubbing the rims of the glasses with their fingers (Figure 15). Certainly this could be accomplished by trial and error. However, students were challenged to look for a process that would be more efficient and precise.

Figure 15. Producing musical tones by rubbing the rim of a glass.

After some thought, they decided to add a specified amount of water to a goblet and measure the frequency that resulted from rubbing its rim. The process would be repeated by adding successive measured volumes of water and measuring the resulting frequency after each addition. Eventually, they used a spreadsheet to create a chart (Figure 16) of frequency versus volume of water in the goblet and fit a polynomial to the data. Since the frequencies of desired notes are known, it was then a simple matter to compute the volume needed to produce a specific note and to quickly prepare a series of goblets that would allow them to play all the notes in the song.

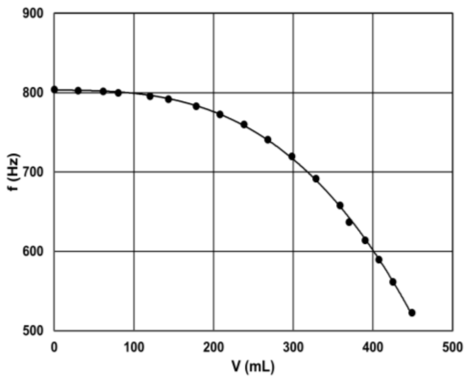


Figure 16. Frequency of produced sound vs volume of water in the glass

Soon after this, students were exposed to how guitar strings play various notes and how the frets are spaced so as to produce the exact frequencies of the notes in the musical scale. To reinforce this new learning, they were asked to create a stringed instrument from a cardboard box and fishing line – using toothpicks to form the frets (Figure 17). The instructor was gratified to see that all of them computed the location of the frets from the formulas we developed and then placed the toothpicks at these exact locations – as opposed to using trial and error.

The engineering design projects described above were used to emphasize that science and mathematics are tools that can be employed in the design process. A more extended and open-ended project was assigned next. For the lesson plan they were following, the engineering challenge was to design a musical non-stringed instrument that would play all notes of a major scale within an octave. One group decided to make a pan flute and brought a box of straws with them to class. They used the guitar string formulas developed in class to determine the required length of the straw representing each note, cut the straws to the required lengths, and banded them together. A scale was played on the first attempt and their expressions of satisfaction, rather than amazement, indicated to the instructor that the notion of mathematics- and science-aided design was sinking in.

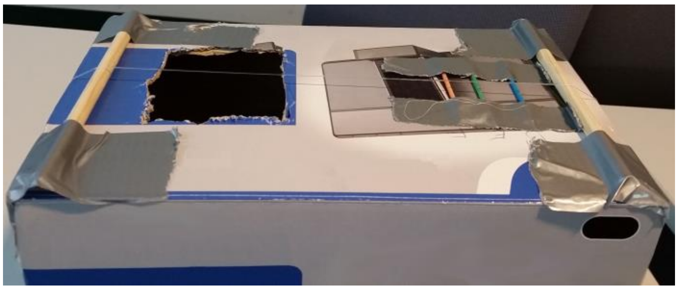


Figure 17. A cardboard box string instrument

Another group decided to make wind chimes out of metal pipe. They too dutifully followed the process of cutting the pipes to different lengths indicated by the frequency-length formulas developed earlier. The resulting chimes had a beautiful tone but, unfortunately, did not play the correct notes. Internet research quickly revealed that the physics of a guitar string and pan flute resulted in the same (inverse) relation between length (of string or air column) and frequency – but that the relationship was different for wind chimes, where an inverse square relation is followed. Students learned earlier that scientific knowledge could play a vital role in the design process. Now they learned that it has to be the right scientific knowledge.

3.2.2.3. Spring, Rubber Band or Air Powered Car

For their integrated STEM lesson plan, one group selected the topic of constructing a car powered by a rubber band, spring, or air pressure (via a balloon). The lesson plan covered several meetings of both a seventh year mathematics course and seventh year science course. Standards based content covered in those classes included kinetic and potential energy, how to design an experiment, dependent and independent variables, plotting, and proportions.

For the culminating design project, students were provided balloons, straws, paper clips, coffee stir sticks, CDs, cardboard, rubber bands, scotch tape, mouse traps, empty water bottles, string, and dowels and tasked to design a vehicle powered by air, elasticity, and/or a spring that would travel a certain specified distance.

This is a fairly standard type of project to assign middle school students and it usually provides an activity where the students can be creative and have fun but does not necessarily require them to use any science and math. The special feature of this project was the constraint that the vehicle travels (within a tolerance) a fixed distance. This one small add-on forces the students to perform experiments (for instance, distance travelled versus windings of the rubber band) and make a graph – so that they would have a calibration for their vehicle. It seems like a minor modification to add this constraint to the design problem but it emphasizes that engineers have to design under constraints and, more importantly, it requires the use of science and mathematics skills as part of the design process.

3.3. PRIMARY SCHOOL

The action learning pedagogy for teaching mathematics through technology-supported real-life projects can be recommended for the primary level as well. Certain differences between that level and the higher levels of mathematics education have to be highlighted. First, the authors will use the descriptor "mini-projects" to describe the context of the action taken at the primary level, based upon their apparent simplicity – in terms of content used, technological skills needed, and societal outcomes produced – as compared to the higher level projects. The reflection component of the action learning pedagogy is fulfilled throughout the mini-project as the teacher (or teacher candidate) asks their students a series of inquiry-based questions about the real-life consistency of their solutions, i.e., "how realistic is this answer?" or "how could we tell if this was the wrong answer?" In addition, the students (as will be shown below) may be asked to pose and solve their own problem similar to a problem offered by the teacher. This kind of reflection stems from the ideas of Freire who argued that "problem-posing education… corresponds to the historical nature of humankind ... for whom looking at the past must only be a means of understanding more dearly what and who they are so that they can more wisely build the future" [27]. This belief is congruent with the action learning pedagogy of mathematics that encourages reflection on what was already experienced through problem solving in order to come up with and then resolve new quires.

3.3.1. Action Learning Pedagogy at the Primary Level

One of the authors supervised mini-projects carried out over the years by teacher candidates in the framework of a capstone experience in mathematics signature pedagogy within a professional development school [41]. The mini-project described below involved 2nd grade students recommended by their classroom teachers for an afterschool activity. Five 50-minute sessions were conducted in a computer lab of a rural elementary school equipped with electronic spreadsheets. One of the greatest facilities of a traditional spreadsheet is that it allows young children to explore the results of simple yet automatic calculations and draw conclusions about the results. Thereby, one can learn mathematical concepts prior to the development of formal procedural skills. For example, the concept of average requires one's proficiency in addition and division. The latter operation is not introduced until the 3rd grade at the earliest. Yet, a spreadsheet can be designed to allow the students, by using the scroll bars attached to the cells, to alter the cell values and interactively display the average rounded to the closest whole number—the only number system that conforms the 2nd grade mathematics curriculum [24].

Prior to the mini-project, the students were not familiar with a spreadsheet and the needed skills that were taught as the activities progressed. They worked individually and were assisted by the teacher candidate, two college of education faculty members, and additional parent volunteers when appropriate. Throughout the project, the students demonstrated a superb on-task behavior that can be ascribed to the enjoyable context of the activities (surface structures of teaching and learning), their natural drive for curiosity, the design of the computational learning environment that included reflection (deep structures of teaching and learning), and the project supervisor belief about what it means to do mathematics at that grade level (implicit structure of teaching). Reflections of young children solicited as part of the mini-project are presented in Figure 19 and Figure 24 below. Reflection of other parties involved in mini-projects are presented at the conclusion of Section 4.

3.3.2. Examples of Primary School Mini-projects

A few examples of mini-projects, designed and supervised by one of the authors, are presented in the sections below.

3.3.2.1. High and Low Temperatures

At the beginning of the mini-project, five temperatures were listed for the week and students were asked to find the highest and lowest temperatures (comparison of numbers by ordering them) and then calculate the temperature range for the week (using subtraction). When using subtraction, students were expected to subtract the smaller number from the larger number so that the range is a positive number. This might be if not the first but at least a true encounter of young children with a physical meaning of a mathematical result. Noting the need to have a positive range prepares them for the future when one has to reject negative areas and volumes resulting from an erroneous integration either by hand or using technology. Also, this skill is critical when designing a spreadsheet to solve the raft problem numerically.

Then the students were asked to use the sliders to decrease the lowest temperature for the week by five degrees, and to find the resulting change in the temperature range (Figure 18). This task also contributes to their appreciation of the fact that by decreasing the lowest temperature, they increase the range. Likewise, increasing the highest temperature affects the temperature range. At the same time, decreasing (increasing) temperature of any day may not result in the change of the temperature range for the week.

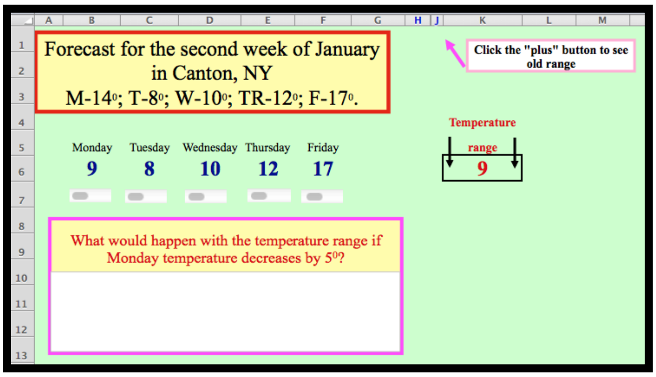


Figure 18. The study of temperature range.

Another notable example of a possibility of disregarding physical meaning of a situation is as follows. A 2nd grader was asked to pose her own problem about temperature range using a spreadsheet and was so excited about the ease with which the program does computations, that she formulated a problem of finding a five day temperature range with the answer 95 degrees. While this problem is numerically coherent (i.e., it has a formal solution), it is not contextually coherent (even when temperature is measured in Fahrenheit) just as the negative values for area and volume have to be rejected based on the physical meaning of these characteristics. This example shows the importance of the teacher's assistance in helping a student to understand the deficiency of the posed problem based on real-life experience. In terms of Gestalt psychology, such uncritical use of technology in posing a problem can be interpreted as the manifestation of Einstellung effect [49] – a tendency of using a workable problem-solving strategy in situations that can either be resolved more efficiently or to which the strategy is not applicable at all. Here, the ease of computing was in contradiction with real-life context within which temperatures while vary, do this variation within a certain range only. In turn, temperature range varies across the whole spectrum of parameters. One might request an explanation of why weather temperatures can only vary within certain limits thus demonstrating how questions seeking explanation differ in complexity from questions seeking information [43]. At the same time, questions of that kind, while cannot be adequately answered in the context of the primary school they do open a window on the future learning of science for the 2nd graders.

After the students used the spreadsheets to construct their charts, they were asked to reflect by describing what their favorite part of the activity was. Two of these reflections are shown in Figure 19.

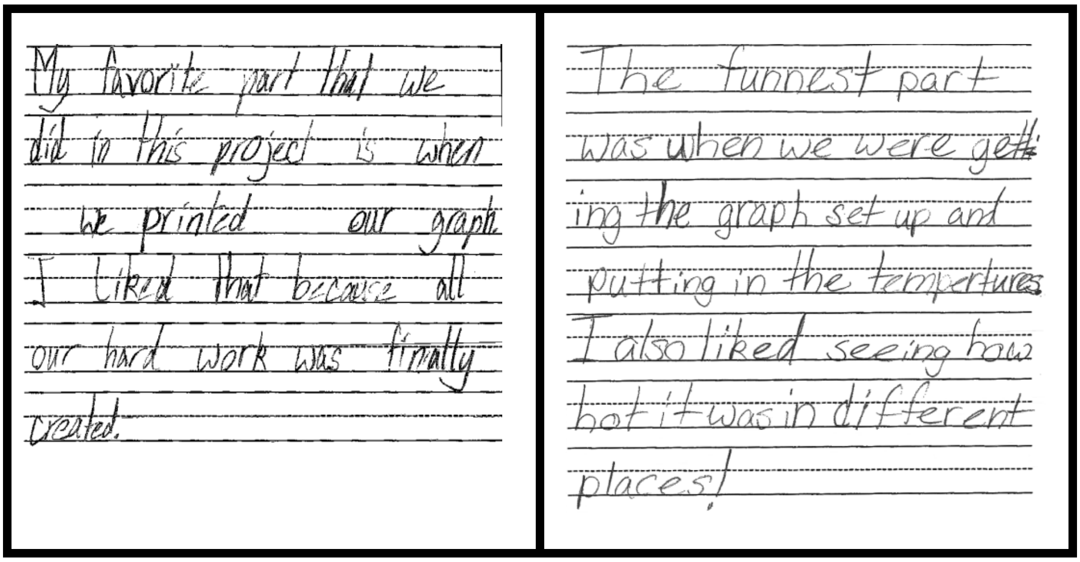


Figure 19. Two reflections on the temperature mini-project by second graders.

Another part of the temperature mini-project is worth noting. The students worked on a task dealing with the concept of average (arithmetic mean) as a characteristic of outdoor temperature change. They had difficulty comprehending the ‘single question—multiple answers’ didactic construct. The students were asked: *What could have happened with the temperature in the duration of five days if the average temperature has increased by one degree?* The response was that there is the only one answer to this question: There was just one day when the temperature went up by five degrees. Without a proper intervention, the students were unable to overcome the concreteness of a single day, not to mention their inability to grasp a possible split of temperature increase over several days.

3.3.2.2. Rings on fingers

A proper intervention unfolded in the form of the following hands-on task: *Find all the ways to put five rings on the index and middle fingers*. Experimentally, using rings, the children found the answer and recorded it in the form of drawings (Figure 20):

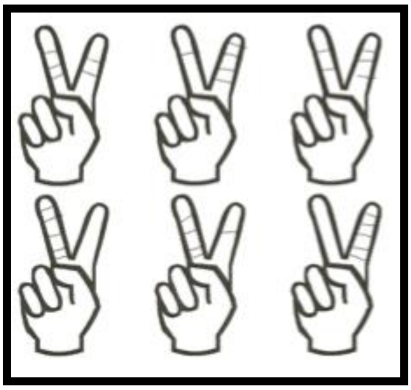


Figure 20. Representing 5 as a sum of two non-negative integers (Source: [3]).

- five rings on the index finger;
- five rings on the middle finger;
- one ring on the index finger, four rings on the middle finger;
- one ring on the index finger, four rings on the middle finger;
- two rings on the index finger, three rings on the middle finger;
- three rings on the index finger, two rings on the middle finger.

Furthermore, they were able to connect the rings activity with the possibility to accumulate the increase of temperature by five degrees over two days. With the appropriate teacher guidance, they constructed an isomorphic model using rings and fingers, recorded this model through a drawing, and then by writing all representations of the number 5 as a sum of two non-negative integers, applied their findings to the temperature change context.

3.3.2.3. Frequency Assignment Problem

Consider Figure 21 in which three same size circles – $C1$, $C2$, $C3$ – are arranged in a way that $C3$ overlaps with $C1$ and $C2$ but $C1$ and $C2$ don't overlap. Other mutual arrangements of three circles in the plane are possible. One may be asked:

How many different ways can three circles be mutually arranged to either overlap or not?

The relation of the above question to the "E" component of STEM is that it has its origin in the following complex radio-engineering problem [7,58]:

Consider a cellular telephone network with radio transmitters, which must be assigned one or more operating frequencies. If two nearby transmitters are operating on the same frequency, they have the potential to interfere with each other. In the simplest model, the frequencies assigned to any such pair of transmitters are required to be distinct, and the objective of an engineer is to minimize the total number of frequencies used within the network.

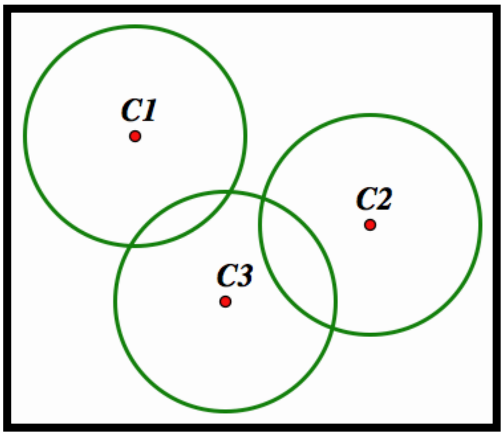


Figure 21. Cellular telephone network with three transmitters.

Several rudiments of the problem can be introduced at different grade levels. For example, kindergarten students can be given physical materials (e.g., either paper or plastic circles) to explore the idea of overlapping/non-overlapping circles and, in doing so, to appreciate the multitude of their mutual arrangements. First and second graders can explore the different arrangements of the centers and radii of the circles using measurements. Third graders can use a dynamic geometry program to move the circles around, developing a hands-on solution to the problem by recording different arrangements of the circles in a chart. Students in grade four can use a tree diagram approach to answer the original question. Fifth graders can be shown how to extend the question to more than three circles, leading to the problem of finding the sum of consecutive counting numbers starting from one (these sums are known as triangular numbers). Finally, students in grade six can be shown the full complexity of the problem by using a spreadsheet as a modeling tool [7].

3.3.2.4. Counting M&Ms

Students were given the following hands-on tasks: Count the number of each color (among red, yellow, green, blue, and orange) in your bags of plain and peanut M&Ms, and represent the numbers found in the form of bar graphs. Eat all M&Ms that do not belong to your data. Use the graphs to decide: (i) which color has the most and the least candies

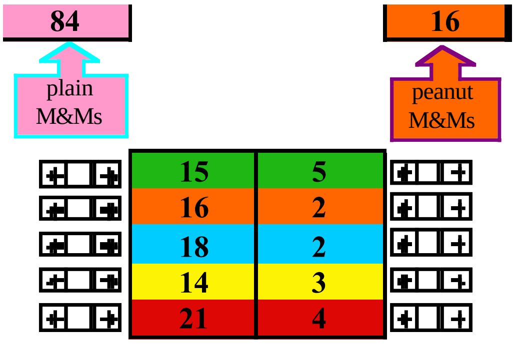


Figure 22: Spreadsheet with sliders to record the counts of plain and peanut M&Ms by color.

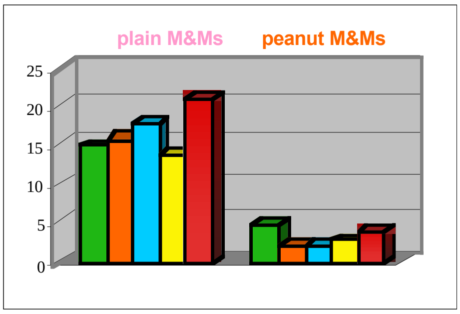


Figure 23. Dynamically updated bar chart representing the counts of the Plain and Peanut M&Ms in Figure 22.

in the plain (peanut) M&Ms? (ii) what combination of two colors when added together have the most and the least candies in the plain (peanut) M&Ms? Note that these tasks were completed as a combination of class and home activities. In particular, the students were not allowed to take their candies home, and the motivational element (i.e., eating candies) was limited to class time only. An example of one student's graph is depicted in Figure 22 and Figure 23.

After the idea of graphing as a visual representation of data collected had been internalized by the students, the introduction of the computer program was highly conducive for the students' further conceptual development. Their ability to manipulate a physical environment (i.e., drawing graphs) was significantly below their intellectual potential to understand mathematical concepts involved. In fact, the design of the computer program was aimed at liberating the students from the tedium of extensive physical manipulation, enabling them to concentrate their attention on mathematical tasks and to work on increasingly difficult problems without being hindered by the lack of sophisticated fine motor skills required.

On the third day the students were very excited about using computers to plug in their numerical data (Figure 22) and to interactively visualize the friendly production of familiar graphs in accustomed colors (Figure 23). The use of spreadsheet sliders incorporated into the computer program made it possible to keep this task at the click-and-see level of physical manipulation. The students remained eager and focused on the task of creating a computer copy of their hand-made graphs, because they knew they would be

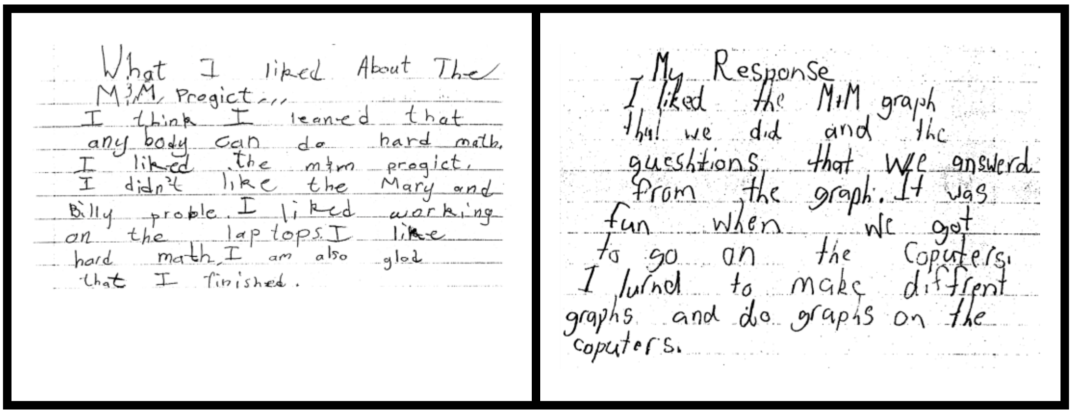


Figure 24. Two reflections on the M&M mini-project by second graders

printing this copy on a color printer and taking it home to share their computer skills with their family. These graphing activities were extended to include the early probability strand when the comparison of what is most probable was made possible by interpreting bar graphs in terms of their height: the taller – the more probable. Figure 24 shows students' reflections on this mini-project, demonstrating how the use of technology allowed them to survive what one of the students referred to as "hard math." This is another example of the importance of integrating action learning pedagogy at the primary level as a way of preparing students to be successful in mathematics at the higher levels.

4. Preparation for Mathematics Educators Teaching Action Learning

Training groups, such as MUG, are important for facilitating the training of mathematics educators. Independent organizations provide a cohesiveness required to integrate mathematics education vertically throughout all the mathematical levels, and horizontally among the other STEM fields and the surrounding community.

Primary	• Inquire-Based Questioning Training • Mentorships and Group Meetings • In-Service Teacher Education
Secondary	• Project Development Workshops • Mentorships and Group Meetings • In-Service Teacher Education
Post-Secondary	• Graduate Student Seminars • Mentorships and Group Meetings • Faculty Development Workshops

Table 4. Proposed structure of educator preparation for teaching mathematics through action learning across all academic levels

4.1. Training for Post-Secondary Faculty and Graduate Assistants

MUG is actively engaged in training graduate students and interested faculty to teach mathematics with action learning.

4.1.1. Student Development

4.1.1.1. Undergraduate Training

MUG collaborated with STEM Mart – the USF multidisciplinary STEM tutoring center – to train dozens (30 per semester) of experienced undergraduate STEM majors as tutors. In particular, tutors were selected on the basis of their performance in the calculus sequence, and a priority was given to students that produced quality interdisciplinary applied calculus projects. In addition to their standard tutoring responsibilities, the STEM Mart tutors received special training in leadership skills and assisting their peers with the project related activities such as sharing their project experiences with their peer, helping them with office software (Word, Excel, etc.), and recommending a subject area advisor.

4.1.1.2. Graduate Student Development

To date, 79 STEM graduate students have been involved as project advisors and 8 in editing submissions for the UJMM. These students are paired with undergraduates, supervise their work, and help them write their final project. This is a unique and unorthodox training that standard mathematics TAs, who only hold help sessions or teach, do not obtain.

4.1.2. Young Faculty Development

Newly hired faculty members can be introduced to the action learning teaching program through the mentorship of an experienced faculty member. Before the semester begins, the faculty mentor should meet with the new instructor, provide her/him with the project and advisor guidelines, discuss how to access the program’s network of advisors, and answer any other questions about the action learning pedagogy. As the semester progresses, the faculty mentor should keep in contact with the new instructor, and answer any follow-up questions that may arise.

Initially, new faculty members are advised to offer the project-option in a single class to a limited number of students, so as not to become overloaded by all of the different fields encountered while advising a multitude of interdisciplinary projects. After teaching with the project-option for a few semesters, the young faculty member should be adequately prepared to teach a project-only section without substantially increasing their overall workload.

4.1.3. In-Service Faculty Development

After each semester, the mathematics faculty members who teach with the action learning pedagogy should meet and reflect on their advising experiences. The mathematical tools that are most frequently used by their students’ projects should be identified, and special emphasis should be placed on those topics in the subsequent semesters. The instructors should also recommend the best projects from their sections for publication in the UJMM.

To disseminate the action learning pedagogy to the surrounding colleges and universities, MUG hosts the biannual Workshop for Interdisciplinary and Inquiry Based Teaching of Calculus that introduces interested faculty to the organization’s activities alongside the STEP Up for Calculus Student Research Poster Conference. The past two workshops were attended by 52 faculty members, representing 11 Florida colleges.

4.2. Training for Secondary School Teacher Candidates

One of the authors is a faculty member in a College of Engineering, but is the instructor for the capstone course of a bachelor degree program that produces middle school science and mathematics teachers – offered through the College of Education. One of the goals of the course is that the students, upon graduating and becoming in-service teachers, are able to include engineering topics and concepts in their lesson plans [15,45,71]. Throughout their degree program, these students have been exposed to science and mathematics content and pedagogy. In the capstone course, they are exposed to how mathematics and science may be integrated with each other and into engineering as well. Since most students are fairly well versed at this point in what scientists do, but know little about what engineers do, a portion of classroom time is spent describing the different goals of scientists and engineers, and covering the different tools and approaches they use to achieve them. The main deliverable for the course is that students (working in groups of two or three) produce an integrated STEM lesson plan. The lesson plan is to focus on a single theme (e.g. weather or conservation of energy), to be several days in duration, and to be shared between a middle school science and mathematics course. The plan is to culminate in an engineering design project.

Before developing their own lesson plan, students work through an existing lesson plan that has been developed by a group of local high school teachers and which has been reviewed by peers, local school district personnel, and the course instructor. In addition, a number of stand-alone hands-on examples, utilizing parts of the engineering design process, are covered by the instructor.

4.3. Training for Primary School Teacher Candidates

It should be noted that teacher candidates currently experience the action-learning pedagogy in the context of a professional development school [41] – enrolled in an internship (pre-student teaching) – in which they supervise afterschool activities for young children. Here is how one teacher candidate reflected on her experience with action learning, which merged into a research project – a typical arrangement within a professional development school:

> "I have been very happy with my decision to do research... it has given me a great experience, uncommon during the MST program and has really helped set me apart during job interviews."

The following reflections by several other parties involved in the projects are indicative of action learning taking place among the members of the community of practice [79].

A teacher candidate [1]:

> "I wouldn't have been able to successfully run the project without it being collaborative… A supportive mentor teacher, a supportive school, and clear directions from the SUNY professors… The school – they have to be willing and almost excited at the prospect of being a host school".

A mentor teacher [1]:

> "Collaboration is always a win-win situation. The students were offered the opportunity to be involved in a technology project. They always looked forward to the after-school sessions and were eager to tackle the project/activity for that day".

A parent volunteer [1]:

> "Students need new and relevant ways to learn math – technology is their passion, so it makes sense to team them up… Ted [her son] had been hesitant about liking math, but loves computer… This new approach pique[d] his interest".

A clinical faculty member involved in one of the mini-projects [61] questioned the rationale for a CASP-related project expressing her belief that school children, when asked to do something, should have a purpose. The search for a purpose led to many new developments in CASP, in particular the introduction of the so-called integrated spreadsheets [61] using videos within a traditional spreadsheet environment, something that became quite popular with the 2nd grade participants.

By carrying out the above-mentioned mini-projects (Section 3.3.2), elementary teacher candidates can develop powerful experiences in teaching mathematics with an engineering focus – that are very different from traditional educational practices with little or no connection to real life – by learning to model mathematical concepts within grade-appropriate learning environments. Just as in the case of secondary school, research shows the importance of incorporating engineering education into the curriculum for elementary teacher candidates in an "ad-hoc" format (Katehi, Pearson, & Feder, 2009; Rogers & Portsmore, 2004). With this in mind, the mini-projects were also discussed within an elementary mathematics content and methods course taught by one of the authors demonstrating real-life applications of mathematical concepts found in the primary school mathematics curriculum.

5. Conclusions

The need for highly trained STEM workers in the United States continues to grow as our society becomes more reliant on science and technology, but the 60% attrition rate among prospective STEM majors [26] is a daunting challenge. The US President's Council of Advisors on Science and Technology has recommended that college-level mathematics be taught "by faculty from mathematics-intensive disciplines other than mathematics" [26], but the American Mathematical Society is "in strong disagreement with these specific recommendations" and asserts "that it is essential that mathematicians be actively engaged in the planning and teaching of the mathematics courses that form the foundation of STEM education" [29]. Rather than supplanting the current mathematics teachers, we propose the adoption of a project-based action learning system, which incorporates technology in a meaningful way, to be the signature pedagogy for K-16 mathematics education.

Starting at primary school, this pedagogy introduces aspects of science, engineering, and technology into the mathematics classroom, while engaging students and teachers on a surface, deep, and implicit structural level. Inquiry-based projects afford students the opportunity to pursue their interest and independently discover the usefulness of mathematical tools. Introducing mathematics as a system of critical thinking to developing creative solutions for real-world problems – rather than a collection of esoteric formulas to be memorized – can create a positive first impression upon a student, which can influence their decision to choose a career in STEM. As one of the prospective elementary teachers put it, "It's easy to memorize formulas and plug numbers in to solve a problem, but understanding why the formula works how it's relevant is a whole other level of understanding. Knowing why a mathematical procedure or formula works and how it's related to 'real world' will not only increase interest in the material itself but will allow students to better process and conceptualize mathematics subject matter". The authors believe that this comment by the teacher is true for higher levels of mathematical learning as well.

As students begin to mature in their STEM education, their interests will begin to diverge, requiring the assistance of subject area specialists. Synergistic collaborations between students, mathematicians, and other STEM professionals encourage students to actively engage in the learning process while receiving directed feedback and guidance.

The action learning pedagogy for mathematics education is actualized at the primary, secondary, and post-secondary levels as congruous, though distinct, methodologies. When used in conjunction, the three action learning pedagogies form a cohesive continuum for all levels of mathematics education. However, the benefits for implementing the pedagogy at one educational level are not contingent on the adoption of the system at all levels. Administrative efforts can either be used to stimulate the interdisciplinary collaboration of the faculty and the surrounding community – facilitating action learning for many classes – or to develop special programs for a limited number of students, e.g., afterschool activities, summer programs, capstone projects, or simply incorporate the pedagogy into other existing programs.

The next generation of K-12 mathematics teachers and university mathematics faculty, i.e., current mathematics education and mathematics graduate students, should be taught mathematics with some interdisciplinary background and examples. All countries need qualified mathematics faculty and teachers who are not looking for an excuse to avoid the K-16 interdisciplinary mathematics applications experience. An effective way to get such an experience is based on advising mathematics application student projects, generously suggested by many volunteers: community professionals and non-mathematics faculty. Such projects should be an important part of the STEM education curricula throughout the world.

6. Acknowledgements

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