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TEACHING MATHEMATICS AT FACULTIES OF ENGINEERING IN BOSNIA AND HERZEGOVINA VIEWED THROUGH TEACHING AND SOLVING EXTREMAL PROBLEMS – A CASE STUDY

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Abstract

This paper analyses the situation in teaching mathematics at faculties of engineering in Bosnia and Herzegovina based on research conducted using a test comprising ten extremal problems administered to second-year students enrolled in the four most popular engineering study programs. The aim of the research was to review the local mathematics teacher training and refine the traditional methodology of teaching mathematics, as well as to analyse teaching the subject to students of engineering, especially their understanding and mastering topics such as differential calculus, more specifically the derivative of functions of one or more variables, and its application in solving extremal problems and developing students' ability to apply the acquired knowledge in learning physics and specific engineering subjects. The obtained results were used to make recommendations as to how to improve the teaching of mathematics at local universities.

Key words: teaching mathematics, mathematical education, Bologna process, extremal problems, application of mathematical knowledge in other sciences, recommendations for improving the teaching process

ZDM Subject Classification: C75, D65 AMS Subject Classification: 97A99, 97A80, 97A30

1. Introduction

The launch of the Bologna Process brought change to universities across the European continent. Although studying in accordance with Bologna is accepted as an indisputable fact, there is still opposition and debating when it comes to this system of education [9, 14, 16]. The Bologna system of studies altered completely the previous system of education in Bosnia and Herzegovina, Serbia, Croatia, as well as other adjacent countries. Even though this system of studies, compounded with the development of technology, has brought many benefits, it also has its downsides. Namely, when it comes to Bosnia and Herzegovina, it was imposed as mandatory and implicit. The system itself requires that a number of prerequisites be fulfilled before it can be implemented successfully. We believe all universities in Bosnia and Herzegovina introduced this new system of studies before they ensured the basic requirements needed for this system of higher education were met, including the corresponding conditions in terms of logistics, space and staff.

Several years after the introduction of this system of studies, there have not been any major improvements in terms of the quality of education. The ―speed" of studies has only contributed to this, since the number of courses taught during one academic year significantly increased with the introduction of one-semester courses, mainly doubling it, and according to students, the time needed to study individual subjects systematically is insufficient. Consequently, students are forced to use a faster approach to studying, which is not necessarily more efficient. The number of students whose sole interest lies in earning a passing mark, without actually understanding or acquiring the subject matter taught, is also on the rise. Furthermore, students' personal interest has decreased, and it is well known that curiosity and interest are the key factors for the success of any teaching/learning [16, 17, 19].

What has been stated above has also had an impact on the teaching of mathematics at universities, especially at faculties of engineering, where mathematics is considered a core subject for learning engineering subjects and the overall education of civil, electrical, mechanical and other engineers. The appreciation of mathematics in technology, economics, natural and social sciences is growing. It is for this reason that one of the more important goals and tasks of the mathematics teaching methodology should be capacitating students to employ the acquired knowledge of mathematics in other disciplines. Courses in mathematics at universities in Bosnia and Herzegovina are generally taught ex-cathedra, and the subject matter presented is traditional and does not vary greatly between departments, at least not when it comes to the first year of studies at faculties of engineering.

The contemporary methodology of mathematics offers various possibilities for introducing pupils and students to individual work and research, developing their ability to solve problems and fostering their creative abilities and their capacity for creative thinking. The focus is placed on developing students' potential to study mathematics independently and in a creative way, as well as on creating preconditions for them to apply the acquired skills and knowledge of mathematics. Student independence in learning and studying mathematics is largely achieved through the adequate selection and use of mathematical problems. Thus, mathematical problems become an important tool for students to acquire the fundamental knowledge of mathematics and establish the necessary skills and habits, enhancing their mathematical abilities and their potential for creative thinking. It may be said that solving any mathematical problem has a revelatory and creative aspect to it. Critical thinking and problem solving go hand in hand. In order to learn mathematics through problem solving, the students must also learn how to think critically [11].

The ability to use mathematics in other disciplines is generally expected of all science and engineering students. Anecdotal evidence suggests that many students lack this ability. Science and engineering degrees typically require students to study mathematics as a subject in its own right, with the expectation that students will be able to use the skills and knowledge acquired from their mathematics courses in other disciplines.

There are various papers the authors of which assume that students have a problem applying mathematics. Gill [4, 5], for example, has studied the problems students of physics and engineering have with mathematics. Jackman et al. [5] report on a project involving assessment tasks designed to improve the ability of students to apply/use mathematics in context. Britton [2] gave reports on the development and piloting of an instrument which can be used to research the ability of students to apply mathematical skills and knowledge to other disciplines. The instrument consists of mathematical problems set in various contexts. All the problems involve exponential and logarithmic functions, and are based on scenarios from physics, microbiology and computer science. Jakimovik [7] has studied problem-solving competences of the first year students at the Pedagogical faculty (prospective elementary school teachers). The diagnostic tests consisted of fifteen math problems covering areas from the first of two compulsory one-semester mathematics courses. She analyzed the results of the students on two problems: a context (textual) problem which can be solved by logical reasoning or by using a proportion and a context problem which can be solved by logical reasoning or by modeling it as one linear equation with one unknown or as a system of two linear equations with two unknowns [6].

There are a few papers, however, which specifically address the question of whether or not university students are able to use mathematical skills and knowledge to other disciplines.

Mathematical knowledge, skills and abilities are needed for all professions; this is particularly true for professions in which exact explanations are needed, and for professions connected to natural sciences, engineering, technology and economics [14, 15].

2. Organization and aims of the research

This paper presents the findings of the research conducted with second-year students of the most popular engineering departments at four universities in Bosnia and Herzegovina during the academic year 2009/10, and examines how mathematics is taught to engineering students. The research conducted allowed the author to observe and analyse the situation at faculties of engineering in terms of teaching mathematics using extremal problems. Due to their extensive application in various fields, extremal problems show how previously acquired theoretical knowledge of mathematics may be used, i.e. how the right branch of mathematics ought to be exploited to deal with specific problems, science and engineering subjects, problems encountered in practice, etc.

The aim of the research was to suggest possibilities for improving and refining the traditional methodology used to teach mathematics, and also to examine how mathematics is taught to engineering students, especially how they understand and master topics such as differential calculus, or more specifically, the derivative of functions of one or more variables, and its application in solving extrema problems and developing the students' ability to apply the acquired knowledge in learning physics and specific engineering subjects.

The tested sample consisted of 236 second-year students of electrical, mechanical and civil engineering were tested, and the results obtained were used to offer solutions to some issues and problems arising nowadays in the teaching of mathematics. These issues included the extent to which the mathematical notions presented to students in the first year have been truly acquired; students' ability to model problems encountered in practice using mathematical tools; the durability and applicability of the knowledge acquired in specific specialist subjects; how students' ability to solve mathematical problems compares to their solving of problems in physics, as well as how their ability to solve mathematical problems compares to their success in solving engineering problems.

This research may be classified as non-experimental descriptive, and the testing technique used falls into descriptive research methodology [13, 18]. The students' knowledge was tested using a ten-problem test. More specifically, the test included: one problem related to the theoretical explication of the notion of derivative; two relatively simple extremal problems; one problem asking to calculate the angle between the tangents of two curves; two geometry problems; two physics problems; two problems related to engineering subjects, where the students were asked to model the problems and solve them using differential calculus (different specialist subjects were selected for the different study programs participating in the research, so the problems were different, i.e. the different study programs were given different problems). The students had two academic hours to do the test. The marking system used was as follows: 0 points were awarded for unsolved problems or incorrectly solved problems; 1 point was awarded for partly solved problems; and 2 points were awarded for correctly solved problems. The research was carried out in the academic year 2009-2010.

Department	No. of students	%
Faculty of Civil Engineering	73	30.93
Faculty of Electrical Engineering	82	34.74
Faculty of Mechanical Engineering	81	34.32
TOTAL:	**236**	**100%**

Table 1. The structure of the sample by the study programs.

The Statistical Package for Social Sciences was used for the statistical processing of the results obtained in the research. Since the findings are non-normal distributed data, the following tests were used in their processing: paired samples T-test, Kruskal Wallis test, Mann-Whitney U test and Spearman's rank correlation coefficient (a non-parametric measure of statistical dependence between two variables) [13].

3. Analysis of the results

In terms of their overall achievement, the students of all the three study programs achieved approximately the same test scores. The achievement percentage for all the study programs was about 30% (Table 1). Also, there is no major statistical discrepancy between the groups when it comes to the overall score.

	Groups			Total
	Civil Eng.	El. Eng.	Mech. Eng.	
Number of cases	73	82	81	236
Mean	5.96	6.1	6	6.02
Std. Error of Mean	0.43	0.41	0.36	0.23
Minimum	0.00	0.00	0.00	0.00
Maximum	15.00	16.00	13.00	16.00
Percentage of Success	**29.79**	**30.49**	**30**	**30.10**
Std. Deviation	3.69	3.70	3.24	3.53

Table 2. The test results.

3.1 Solving mathematical problems

Although the first six problems were mathematical problems, they were nonetheless divided into two groups:

1) Group one – problems 1 through 4 (considered easier) – lower-level **MMNN**
2) Group two – problems 5 and 6 (geometry problems) – higher-level **MMVN**

	Points in the first four tasks (problems)		
Number of task	0	1	2
1	18.2%	30.1%	51.7%
2	11.7%	30.9%	57.6%
3	49.2%	19.9%	30.9%
4	70.3%	12.3%	17.4%

Chart 1. Percentage of students who solved the first four problems.

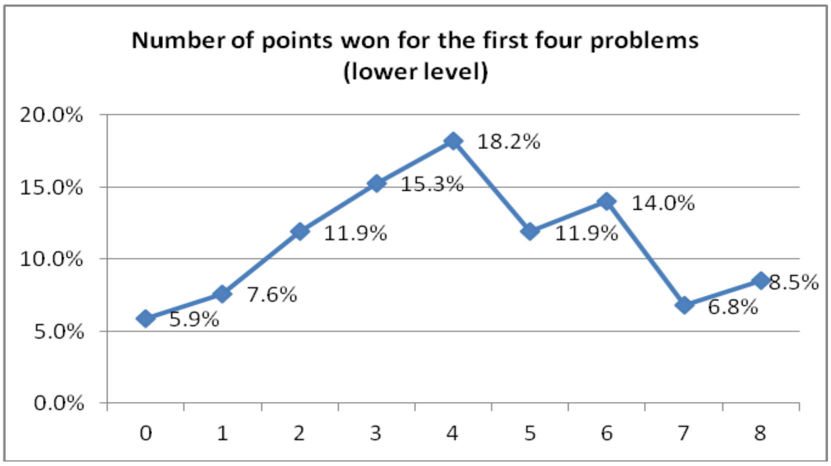

Chart 2. Student achievement for the first four mathematical problems.

Test Statistics [a, b]

	MMNN	MMVN
Chi-Square	2.000	11.405
df	2	2
Asymp. Sig.	.368	.003

a. Kruskal Wallis Test

b. Grouping Variable: STUDY PROG

Table 3. Kruskal Wallis Test.

The students had the highest score in solving the first four problems, classified as lower-difficulty extremal problems, which were labeled lower-level problems (51% achievement). However, even in the case of these problems, the only satisfying achievement was for the first two problems, of which the first was theoretical, and in the second the students needed to determine the local extrema of the given function.

Moreover, although the majority of students were able to give a theoretical explication for the geometric interpretation of the first derivative of a function, when it came to Problem 4 (finding the angle of intersection between two curves), they showed they did not know how to apply the mathematical theory. The mechanical engineering students were the most successful at solving this problem: 28.4% of the students solved it correctly; that is, out of the total number of students who solved Problem 4 correctly, 56.1% were mechanical engineering students, 33.6% civil engineering students, and 10.3% electrical engineering students. Chart 2 shows that the greatest number of students, 43 or 18.20%, won four points after doing the first four problems, whereas only 14 students, or 5.90%, won zero points. All four problems were solved correctly by a total of twenty students, or 8.50%.

Speaking statistically, there is no considerable difference between the student groups, i.e. the three study programs, when it comes to those problems which were easier, or at the lower difficulty level, which means the first four problems (Kruskal Wallis test, Table 3). However, the Kruskal Wallis test showed that, again in terms of statistics, there is a considerable difference between the groups when it comes to solving problem 4 (Table 4). The Mann-Whitney U test was used for additional analysis and confirmed the above finding, showing a discrepancy between the mechanical engineering students and electrical engineering students, as well as as between the civil engineering students and electrical engineering students. The students of electrical engineering scored the lowest in doing Problem 4.

Test Statistics [a ,b]

	PROBLEM 4
Chi-Square	8.083
df	2
Asymp. Sig.	.01

a. Kruskal Wallis Test

b. Grouping Variable: STUDY PRO

Table 4. Kruskal Wallis Test.

In terms of statistics, no considerable difference between the students of the three study programs was observed with regards to their overall achievement after doing the first four problems. This finding is attributable to the fact that the number of students whose solution of

Problem 4 was fully correct was small in comparison with the number of students who solved correctly the first three problems in the given set of problems.

The next two problems were geometry problems:

In the case of Problem 5, the students were supposed to express the area of the rectangle as a function of one variable, and then find its maximum value ($P = x \cdot \sqrt{64 - x^2}$, where x is the base of the rectangle).

Problem 6 also contained the instruction intended for the students to form a function of two variables, after which they were supposed to find the extremum of the formed function.

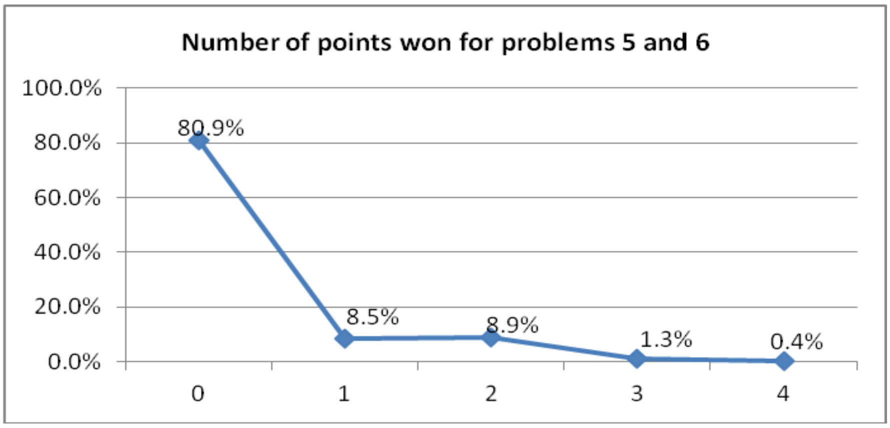

Chart 3. Achievement in solving geometry problems.

To solve Problems 5 and 6, which were geometry problems, the students were expected to use their mathematical modeling skills. The achievement percentage in solving these problems was 7.9%. As many as 80.9% of the students won 0 points for both problems, and only one student (electrical engineering) solved both problems correctly. Statistically, a substantial discrepancy was found between the different study groups when it comes to solving these problems. The students of mechanical engineering achieved a relatively lower score than the others.

Based on these achievement scores, it may be concluded the students do not have enough skills to model geometry problems using mathematical tools, especially when it comes to applying differential calculus and finding the extrema for functions of multiple variables. The author believes it would be interesting to examine further the actual reasons for the discrepancy observed in the level of knowledge between the different study programs. The research findings indicate that this problem may be attributable to an insufficient level of knowledge of mathematics acquired in secondary school, an inappropriate methodology used to teach mathematics at university, inadequate university admission tests, etc.

3.2 Solving physics problems

In solving these problems the students once again demonstrated insufficient ability to model physics problems using mathematical tools. In terms of statistics, the Kruskal Wallis test at the significance level of 0.05 (Table 5) showed the existence of a considerable discrepancy between the different groups in solving the second physics problem, i.e. problem 8, and the Mann-Whitney U test was also used and confirmed this discrepancy between mechanical engineering and electrical engineering students, as well as between civil engineering and electrical engineering students. Electrical engineering students were better at solving Problem 8.

Based on Chart 4, which shows the total points won for solving physics problems, it may be concluded that the students scored rather low. As many as 65.30% of the students won 0 points, and only one student (electrical engineering) solved both physics problems correctly. The first physics problem, Problem 7, was solved correctly by 64 students or 27.1%; the second one—by 6 students, which makes 2.5% of the total number of the tested students.

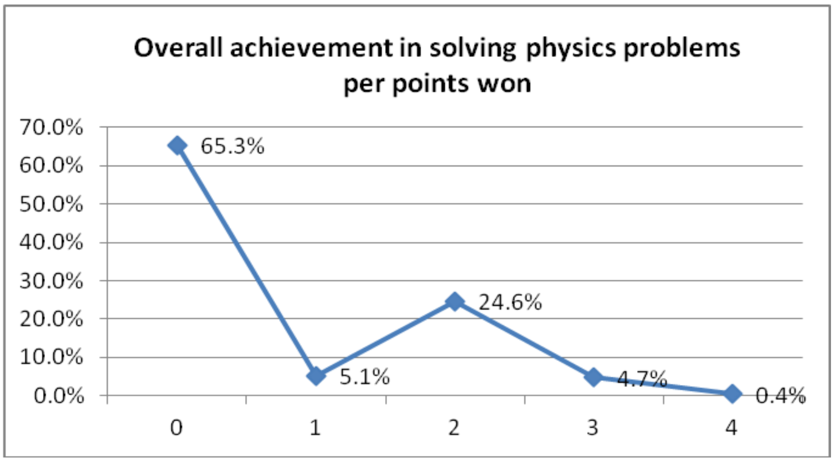

Chart 4. Achievement in solving physics problems.

Test Statistics [a, b]

	PHYSICS1	PHYSICS2
Chi-Square	4.115	10.676
df	2	2
Asymp. Sig.	.128	.005

a. Kruskal Wallis Test

b. Grouping Variable: STUDY

Table 5. Kruskal Wallis Test.

3.3 Solving engineering problems

The problems related to engineering subjects were different for each study program. The students of civil and mechanical engineering were asked to solve extremal problems falling in the field of engineering mechanics; students of electrical engineering were given problems from the fundamentals of electrical engineering.

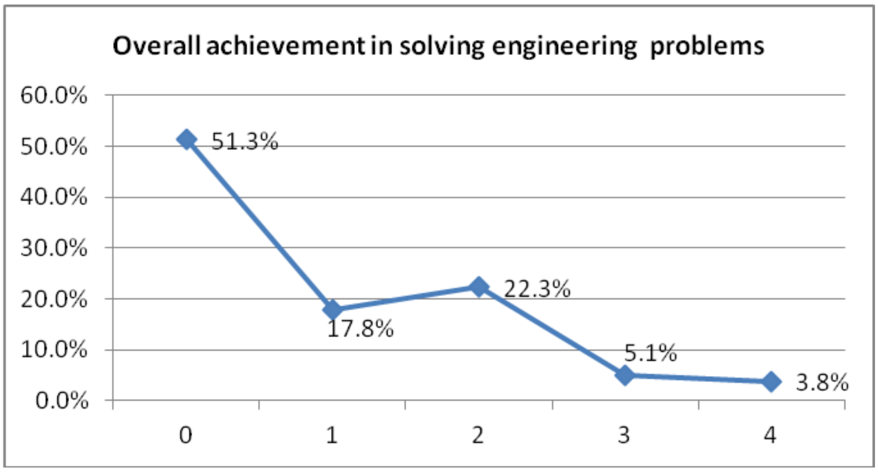

Chart 5. Achievement in solving engineering problems.

Test Statistics [a, b]

	PROBL 9	PROBL 10
Chi-Square	6.783	4.119
df	2	2
Asymp. Sig.	.034	.127

a. Kruskal Wallis Test

b. Grouping Variable: STUDY PROG

Table 6. Kruskal Wallis Test.

It is possible to conclude that the students were slightly better at solving specialist subject problems than at solving physics problems. Chart 5 shows that 121 students, or 51.30%, won 0 points for both problems; 52 students, or 22%, won 2 points, and 9 students, or 3.8%, won the maximum of 4 points.

The results of the Kruskal Wallis test at the level of significance of 0.05 show that the students' ability to solve Problem 9 corresponded directly with their study program (Table 6). As this problem was different for the different study programs, this finding may be taken as a provisional indicator inviting further research. Nevertheless, if the students tested are taken as a whole, it may be said they are unable to employ the gained knowledge of mathematics to deal with specialist problems (only 28 students, or 11.9%, were successful at solving this problem fully correctly).

The students' overall achievement in solving Problem 10 was somewhat higher, although only 21.6% of them solved this problem absolutely correctly. Table 6 shows that there was no interdependence between solving Problem 10 and the students' study program. The thing common for Problem 10 (for each study program) was that the functions were given and the students were expected to use the derivatives to find the extrema of the functions.

Spearman's rank correlation coefficient shows a weak positive correlation ($r_s = 0.231$) between the students' success in solving mathematics problems and their ability to solve physics problems, as well as a medium positive correlation between their ability to solve mathematical problems and their success in solving engineering problems ($r_s = 0.346$). The students who are good at mathematics are also better at physics and engineering subjects. There is also a medium positive correlation ($r_s = 0.381$) between their ability to solve physics problems and problems related to engineering fields.

4. Some recommendations for improving the teaching of mathematics

It is necessary to step up the quality of cooperation between mathematics teachers and teachers of specialist subjects in engineering and other departments. Also, it is necessary to involve the above-mentioned teachers in the preparation of mathematics curricula and syllabi at universities, and also to involve mathematics teachers in the preparation of curricula and syllabi for courses in other disciplines. By doing so, the subject matter of mathematics would relate more strongly to the subject matter taught in other disciplines.

The teaching of mathematics at universities should be organised in such a way as to combine a variety of methods: frontal teaching (where the teacher is the central figure), as well as problem-based, project-based and interactive teaching and learning, where selected themes would be taught with the help of computers, or selected problems would be solved using computers [9, 11, 14, 17].

In discussion groups students should be given more problem-based tasks (or they should do them with the teacher's assistance), such as extremal problems; they should be instructed on how to use the knowledge of mathematics to solve problems in geometry, other disciplines and/or specialist subjects, in order to increase their interest in mathematics and enhance their motivation to extend their knowledge of mathematics. It is necessary to develop students' ability to think critically and model problems in other disciplines using mathematics as a tool.

Complex problems should be solved in several different ways. The famous American mathematician and methodologist George Pólya, originally from Hungary, said a half a century ago: ―It is more useful to solve one problem independently than replicate a hundred problems. It is better to solve one mathematical problem in several ways using different methods, than use a single method to solve many problems." [1, p. 27]. Pupils and students are best motivated if they are successful at solving problem-based tasks independently, and extremal problems may certainly be classified as such.

The curricula and syllabi of mathematics taught at engineering departments in Bosnia and Herzegovina should be harmonised in line with memoranda of cooperation signed by national and international universities. A revision of university programs is necessary in the areas of mathematics, information technology and technical-technology profiles [14]. In particular, it is important that mathematicians become involved in mathematics education research [9, 14].

It is commonly thought by teachers and assistants at universities that the quality of knowledge gained in secondary school is not sufficient to take students to the next level, especially in the light of strict Bologna criteria, which additionally aggravates the teaching and acquisition of mathematics at universities. Therefore, a reform should be launched to improve the teaching of mathematics in secondary schools, which is one of the basic preconditions to implement the above-made recommendations.

It is recommended that all technical departments introduce the following into their work:

1) Preparation of students prior to enrolment in university (preparatory courses)
2) Introduction of a course in the fundamentals of mathematics in the first semester of studies.

5. Conclusion

Based on material presented in this paper, it may be concluded that more than half of the students from the three study programs participating in the research were successful at solving only the first two problems, which were the easiest, practically elementary problems, whereas their achievement was low in solving all other problems. It may also be said that the transfer of knowledge from one field into another, more specifically, application of mathematics to physics and engineering disciplines is not satisfactory, nor is the students' ability to model or express mathematically the various kinds of problems they encounter in other disciplines.

In general, it is possible to conclude there are serious problems to learning mathematics at faculties of engineering, and the durability of knowledge and the students' ability to employ what they have been taught are also problematic.

The conclusion following the analysis of the results obtained in this research is this: the quality of the mathematical subject matter and topics as set by the curricula and syllabi and taught in engineering departments is unsatisfactory. Knowledge quality implies, among other things, the acquisition of subject matter and notions, the interconnection of various notions and topics, the durability of mathematical knowledge, the students' ability to employ the knowledge gained in other disciplines, etc.

Our findings confirm the results obtained in [2, 4, 5, 7].

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Appendix: Test questions

1. Answer these questions:
   a) What is the geometrical interpretation (meaning) of the first derivative of a function of one variable?
   b) How can the extreme values of a function of one variable be found using a derivative?

2. Find the local extrema of the function:
   $$y=\frac{x^3}{3}-5\frac{x^2}{2}+6x+4$$

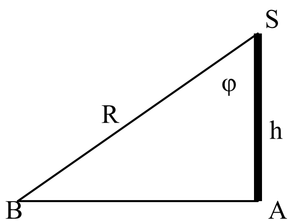
3. Find real numbers *a* and *b* so the maximum value of the function $f(x)=\dfrac{1}{x^2+ax+b}$ is located at the point $T\left(\dfrac{3}{2},-4\right)$.
4. Find the angle of intersection of two curves, $y=\sin x$ and $y=\cos x$, $x\in[0,\pi]$.
5. Inscribe a rectangle of the maximum area into a circle with the radius $r = 4$ cm. Find the dimensions of the rectangle and its maximum area:

a) without using derivatives;
b) by using derivatives.

6. Among all rectangular parallelepipeds with the given volume $V = 8$, find the dimensions of the one with the smallest area. (Note: form a function of two variables and find its extremum).

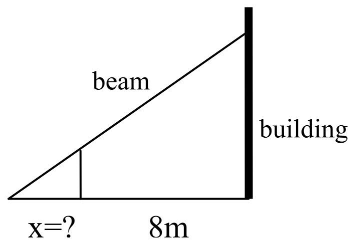
7. An object is ejected vertically upwards from a 10-meter height at the initial speed of $20\frac{m}{s}$. What height does the object reach after t seconds? How many seconds does it take for the object to reach the maximum height, and what is this height? (The height, i.e. the coordinate at moment t for oblique ejection: $y = v_0 \cdot t \sin\varphi - \frac{1}{2}gt^2$, constant gravitational acceleration $g = 9.81$).

8. A light bulb S is positioned perpendicular to point A lying in the horizontal plane of a table. At what height $h = \overline{SA}$ above the table should the light bulb be positioned to secure maximum illumination at point B on the table, if $\overline{AB} = 1.5\ m$? (illumination E at point B: $E = \frac{I \cdot \cos\varphi}{R^2}$).



The following problems were given to the civil engineering students:

9. A 3-meter-tall wall was erected 8 meters from a building (of sufficient height). Determine the length of the shortest rigid beam which can be used to reach the wall of the building from the ground behind the separately erected wall.



10. The bending moment equation $M_x(z) = \frac{p_0 \cdot l}{6} \cdot z - \frac{p_0}{2l} \cdot \frac{z^3}{3}$ is given for a rigid beam of the span l, where p_0 is intensity of load, z is the first coordinates of points on the beam axis. Find the maximum value of the moment ($p_0, l - const$).

These problems were given to the mechanical engineering students:

9. The motion of a physical point is determined by the basic kinematic equations $x = 8t - 4t^2$ and $y = 6t - 3t^2$, where the x and y values are given in meters, and the value of t in seconds. Find the equation for the trajectory of the physical point, the resulting speed v and the resulting acceleration a. What is the initial velocity v_0 of the physical point?

10. The same as for the civil engineering group.

The following problems were given to the electrical engineering students:

9. A resistor with an unknown resistance X is connected in parallel to a resistor of $100\,\Omega$. Both resistors are connected to a $100V$ electrical power source, whose internal resistance is $20\,\Omega$. Find the value of X in a situation where the electric potential difference of the resistor is maximal.
10. The same galvanic elements N = 12 may be used in different ways to assemble a battery, by connecting n elements into a series, and then by obtaining $\frac{N}{n}$ groups in parallel. $I = \dfrac{N \cdot n \cdot E}{NR + n^2 \cdot r}$ is the formula used to show the electricity produced by one such battery, where E is the electric potential difference of one element, $r = 1.5\Omega$ its internal resistance, and $R = 2\Omega$ its external resistance. Find the value of n producing the highest voltage electricity in the battery.