Applying ICT in the teaching of mathematics in high school

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Abstract. Technology is all around us, so why not use it in the education process. We give an overview of ICT applied in teaching of mathematics in high school *"Nikola Tesla"* in Niš, Serbia.

Keywords*: ICT in teaching, e-learning, GeoGebra, Wolfram Alpha, Moodle*

1. INTRODUCTION

In the 21st century there are new, more sophisticated than before, technical devices that are available to everyone, especially to young people. We will single out computers, tablets and smart phones. They are easy to use, and their price is acceptable to all. Young people are growing up in digital environment, using most of these devices with the constant availability of the Internet. Working environment is rich with technical devices of different levels of complexity. Nobody can predict with certainty the expansiveness of technological development as well as the direction in which we will advance the most.

Technology is all around us, so why not use it in the educational process.
Our goal is to have educated students, ready for changes in their environment and prepared for the concept of lifelong learning. We are committed to educating our students to use the devices of high level of technical complexity. Having introduced ICT in the educational process, we will make it easier to our students to cope with the work environment that awaits them after graduation. Double is the benefit: students learn and can immediately check their achievements; on the other hand, they are able to learn about the innovations in ICT in the right place, at school. They can use the information in their work and further learning.

In this paper we give an overview of informational technologies applied in the teaching of mathematics in the Electro-technical school "*Nikola Tesla*" in Niš, Serbia.

2. CURRICULUM

In formal education, a curriculum defines instructional content, materials, resources and means with which students will interact for the purpose of achieving identified educational outcomes.

Electro-technical school *"Nikola Tesla"* in Niš educates six different educational profiles, in the four-year period. Regular program includes *Electrical technician for computers*, *Electrical technician for electronics*, *Electrical technician for thermal and cooling devices* and *Electrical technician for energetics*. In the last seven years, as experimental program, *Electrical technician for information technology* and *Mechatronics technician*, have been added. Mechatronics entered the regular program under the *Rules of the curriculum of the experimental education of the educational profile of Mechatronics Technician* [1], school 2014/2015.

Experimental classes have different programs for mathematics. As the regular program, mathematics is taught for four classes a week, in all four years. Experimental classes have four classes per week in the first and second year, and three classes of mathematics a week in the third and the fourth year. The programs compose same topics, but the topics are differently arranged time-wise in the regular and the experimental classes (Tables 1 and 2). In addition, learning outcomes are not the same for the two types of classes.

Regular programs M12 and M13 are defined by *the Curriculum of education for the common courses in vocational and art schools* [2], which, with the minor changes, has been applied for nearly 25 years! Experimental classes work under the *Rules of the curriculum for the educational profile of mechatronics technician* [3].

We give a comparative table about some teaching topics and units of mathematics, as well as the number of classes where these topics are dealt with.

Table 1. Teaching units in mathematics for regular and experimental classes

Teaching unit	Regular classes		Experimental classes	
	grade	Number of classes	grade	Number of classes
Mathematical logic and set theory	I	20	II	20
Functions	I		II	
Geometry-axioms	I	12	/	/
Isometric transformations	I	28	/	/
Trigonometry (definition, right-angled triangle)	I	5	I	30
Trigonometry (trigonometric circle, functions, Sine and Cosine Low)	II	52	I	
Trigonometry (addition formulas, equations and inequalities)	II		II	30
Quadratic equations, Complex numbers (introduction)	II	52	I	20
Quadratic equations, functions, inequalities	II		II	32
Analytical geometry	III	48	III	38
Derivative	IV	45	IV	30
Integrals	IV	30	IV	15

Students of the experimental classes can choose one of the optional subjects, with a number of two classes per week. The condition for the formation of the department is having eight interested students. Students prefer some of the professional subjects as their electives. But electives over the last three years, there has been a surprising increase of interest in mathematics as an elective, which the authors interpret as students' increased interest in the further education at the faculties of technical disciplines the recognition by students and their parents of the need for greater mathematical knowledge which should be acquired during the secondary education.

Table 2. Elective courses in mathematics

Elective courses in mathematics	Mechatronics	Information Technologies
III grade	Geometry	Algebra and Analytical geometry
IV grade	Introduction to Analysis	Differential and Integral calculus, Differential equations

Differences in the curriculum for the regular and the experimental classes in a professional, methodical, and didactic terms are obvious. The greatest differences are in the outcomes, and mode (the preferred method, a form of labor, materials). In the experimental classes, the use of ICT in teaching is implied, both by teachers in these classes, as well as in the individual work of students.

We have noticed that there is a gap between the programs M12 and M13 for the regular classes on the one hand and opportunities provided by ICT technology on the other hand. Consequently, we have tried to lessen these differences. It requires the teacher's own initiative for the introduction of changes in methodical and methodological sense. Changes are expected in the future in accordance with the *Guidelines for the introduction of ICT into teaching* [4].
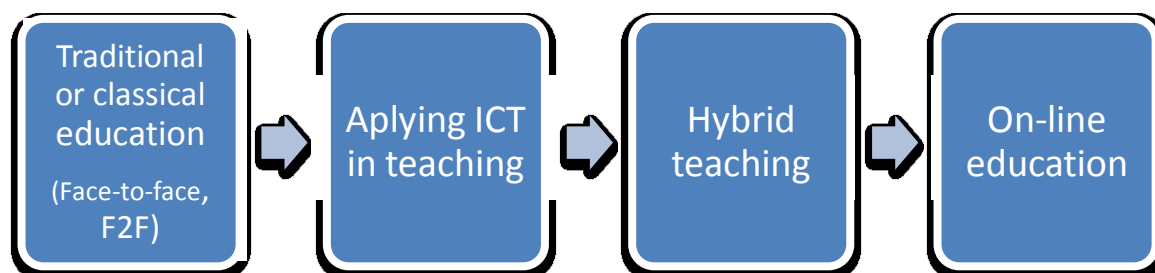
A number of classes generate a "depth" of the intervention in the matter. As the numbers of classes for the same teaching units are different for the regular and the experimental classes, it is natural that the expected outcomes of teaching units are different. For the experimental classes the outcomes are conceptual understanding and the possibility of applicability to specific problems of a technical nature, while in the regular classes much more systematic and synthetic approach is expected. From the experimental classes independent research, project work, teamwork, etc., are the expected outcomes.

3. E-LEARNING

3.1. Stages of e-learning

Electro-technical school *"Nikola Tesla"* for years has been systematically equipping its workspace, so that now there are seven computer classrooms with 12 to 20 new performance computers and video projectors, and two multimedia classrooms (video projector and computer). The multimedia classrooms are available to all teachers. The cabinets that are fitted for electrical specialists' items, also have a computer and video projector, so we can say that the ETS *"Nikola Tesla"* in Niš very well equipped with ICT devices.

By equipping the computer classrooms, the introduction of ICT in the learning process of mathematics is enabled. During the last ten years of work at the school, we have been introducing computers in the learning process of mathematics. First, it started with PPT presentations in classes, and after 2005 *Geogebra* has been introduced as applicative software. With Web 2.0 tools we have continued to digitize the teaching of mathematics.

Traditional or classical education (Face-to-face, F2F)	→	Aplying ICT in teaching	→	Hybrid teaching	→	On-line education

As one can see, at the left end of the above diagram is traditional or classical education (Face-to-face, F2F), which means that learning in the classroom is face-to-face. Technology is not used, except, for example, a word processor to prepare lessons. In the teaching of mathematics in Serbia and elsewhere in the Balkans this method of teaching is almost exclusively used.

The shift in e-education begins by applying information and communication technologies in F2F teaching, mainly in order to improve the classical teaching. This is the lowest, most primitive form of e-education and it represents, for example, the use of Power Point presentations in teaching.

The next phase of the hybrid classes or mixed approach to learning is a combination of the first two forms (classical teaching and learning by using ICT).

The last phase is on-line education as a fully independent, the highest level of e-education. Learning and teaching is carried out only with the help of technology - no F2F classes.

The process of gradual introduction of ICT in schools is in progress, through a variety of system solutions and education (the process of introducing a school-training schools, *Instructions on the program on the implementation of teaching approaches aimed at learning and competence development of students in primary and secondary education and the development of schools*, [5]). It is expected that next year there will be significant changes in the school system, both in the field of on-line learning, and on the level,

"Teaching approaches aimed at learning and competence development of students (hereinafter: the new approach to teaching) in terms of the regulations, includes: planning, organizing and conducting classes focused on learning; learning that takes place largely through curricular and extra-curricular activities in school as a stimulating environment; learning as a process, not a product classes; learning in which students are the subjects of the learning process; learning aimed at developing competence and achievement of outcomes and standards of student achievement; study whose results are constantly monitored and evaluated." (Excerpt from [5]).

However it should be noted that the primary and secondary schools are **educational institutions**. In on-line teaching the educational component of the teaching process is lost. Therefore, we believe that the application of only on-line classes is not suitable for these age levels. We use **hybrid classes** - a combination of traditional classroom and online education.

3.2. Asynchronous and synchronous learning

E-learning, in relation to different ways of presenting content, media formats and modes of delivery of content can be classified into two categories: asynchronous and synchronous. As their names imply, the two categories of e-learning are characterized by the nature of the interaction between teachers and students.

In **asynchronous** [6] modes of learning teacher's role is static, because it sets the electronic materials on the web, and students access at a time that is convenient to them. Interaction between teachers and students can be through questionnaires, e-mails, forums, work assignments or on-line knowledge test (uncontrolled conditions), where the student can

get results immediately (tests) or after testing, assessment and evaluation of teachers. Asynchronous learning recognizes individual differences of students and it allows a student with greater prior knowledge to make faster progress. A very important feature of asynchronous learning is that it enables complete actuality, because the old information can be quickly updated or replaced by the new one.

With **synchronous** [7] learning modes, teacher and students interact directly. Here it is possible to obtain feedback on the spot. Thus, synchronous learning is a form of learning in a real time. All students attending a particular course at the same time participate in the class, regardless of where they are physically located at the moment. This type of learning is achieved through the Internet chat rooms (chat room), teleconferencing, video conferencing and audio conferencing. In recent years, the number of software tools which by the text include voice and video communications is increasing. Videoconferencing is an opportunity for a teaching that involves a teacher and a student who are geographically distant. ETS *"Nikola Tesla"* has already taken out video conference classes, in cooperation with schools of similar educational direction. It is expected that this practice will continue on a larger scale.

3.3. Teaching materials and Instructional design

High-quality teaching materials obtain an increasingly important role in creating courses that may be very different in their methods of teaching, learning objectives, a medium that is distributed, etc. One should follow the *Recommendations of instructional design* [8], that correspond to the target group which they are designed for, intended learning outcomes, and teaching methods. The teachers of ETS *"Nikola Tesla"* use teaching materials prepared by themselves or in cooperation with teachers who teach the same subject, and they rarely use materials and simulations that are available on the Web.

Instructional Design [9] is a theory that defines an explicit reference guide that facilitates the learning process. It lays down the methods and instructions which support and facilitate learning, as well as the situations in which certain methods should be used. In that way, it facilitates the management [10] of teaching. The methods can differ very much one from another, from the previously applied deterministic approach, to the method of trial and error in problem solving.

Behavioral [11], cognitive [12] and constructivist theories [13] of learning have a major impact on the development of modern pedagogical models of electronic teaching and instructional design. We will consider some of these recommendations relating to e-learning [14].

The theory of **behaviorism** [15], considers learning as a conditional adoption of new knowledge, while insisting on measurable learning outcomes. Recommendations of behaviorism can be successfully applied in making paper textbooks and design to the traditional classroom and in making educational and teaching materials for e-learning. Here are some important recommendations of behaviorist learning:

- *Students must be informed clearly with defined learning outcomes* (e.g. "When you learn this lesson, you will know ..., you will understand the connection between ..., you will be able to create ..."). Systematization of knowledge and the outcome of such a definition make it easier for students to control their own learning and provide self-assessment of the learning outcomes (*which are clearly and precisely defined*).

- *Organization and presentation of teaching materials within the lessons, the lessons themselves or part of the course, must have a logical sequence*, for example, adoption records from easier to more difficult one, or from known to unknown terms, from theory to practice, and so on.

- *It is necessary to test the students after completion of the lesson to determine whether the defined learning outcomes are actually achieved*. These results are used in

providing feedback to the student. (In practice, this is not usually feasible, but needs to be tested or test their knowledge after some logical unit, for example, upon the completion of the topic.)

- *It is necessary to provide students with feedback that will enable them to develop and plan their future learning.* (A very important aspect of the learning process is the evaluation by teachers of the results achieved by the students).

Cognitive pedagogy [16] is based on cognitive psychology that examines learning as a mental process that involves memory, thinking, abstraction, motivation and meta-cognition. The influence of cognitive psychology on instructional design and preparation of materials in electronic form reflects the recommendations:

- *Using strategies that increase student attention and enhance perception.* Information should be accommodated in the middle of the screen, the key information should be specifically highlighted (using colors, change the size of text, use graphic elements, etc.), information which you can see on the screen should be limited, it is important to respect the trail information and group it into logical sections. There is no need to introduce elements that distract attention (sound effects, too many animations).
- *The complexity and weight of the material must match the cognitive level of development of students.* One must adapt the material to students with different levels of prior knowledge and enable them to advance their own complexity and pace in mastering the given outcome. It is necessary to establish a link with similar materials of different levels.

3.4. Role of a teacher

In the process of e-learning, a teacher plays the role of mentor, whose task is not to retell the lessons that are interdisciplinary, or even in the form of electronic games and simulations available to all students. Rather, a teacher must follow the development of each student and send him or her some additional sources of information or review opinions and attitudes. Following the exchange of opinions among students and moderating their discussion, or exchanging questions and answers with each students individually, the teacher guides the students in the learning process.

4. SOFTWARE PACKAGES, SERVICES AND PLATFORMS

The most widely used software tools in the teaching of mathematics in the ETS *"Nikola Tesla"* in Niš are *GeoGebra* [17], *WolframAlpha* [18] and *Moodle* [19].

4.1. *GeoGebra*

GeoGebra is an open source is a multi-platform mathematics software [20]. In *GeoGebra* one can create dynamic applets in all areas of mathematics, particularly in the areas of geometry (planimetry, stereometry, analytical geometry). It is possible to use the portable version, without installation, and applets can work on the Web. It can also be used on mobile devices (App for smartphones and tablets). In early September 2014, a new version that supports the 3D view was released. On the website *GeoGebra* [21] one can find a variety of teaching materials. Furthermore, every teacher can create his or her own teaching materials.

In a study (*Test* [22]) and the paper *Influence of intelligence and perception on achievement in mathematics* [23] it was found that students' ability of visualization is inadequate to successfully follow the teaching process. Students are getting worse in geometry, particularly in solid geometry. One of the reasons is the great attachment of today's youth for screens (TV, computers, tablets and mobile phones), and the reality is pre-

dimensioned from the 3D to the 2D! That is why we introduced *GeoGebra* in the process of teaching. It allows students to develop better perception of mathematical content, and, therefore, the application of mathematics and its learning outcomes in other cases. In a similar way, acquired lessons and knowledge can be applied to solving problems in real life.

Students themselves do not have to visualize a problem. The teacher gives the dynamic model, and thus shortens the length of the thinking process required for designing solutions. Students much easier and faster come to the necessary conclusions, since the dynamism of a *GeoGebra* applet indicates the way of coming to solution on their own, as well as for given and similar problems. Adequate visualization in the classes contributed to a better understanding of the procedures and processes, and it enabled the students to better prepare themselves for problems in which mathematics is used (for example, for problems in electronics).

The first time *GeoGebra* (beginning of 2005) was used for the dynamic presentation of prepared teaching materials, but soon we switched to a different use of the software package. After a brief introduction to *GeoGebra*, students have started independently solving mathematical problems of suitable content (trigonometric functions, quadratic functions, analytical geometry, etc.). Students nicely accepted a new methodical approach. *GeoGebra* is used in schools, both in classes and in extracurricular activities (a stand for *GeoGebra* at the "Science is not a bogey" event, *GeoGebra* workshops for all interested students, Open doors of schools, School presentations at meetings, etc.).

As we had the opportunity to use computer classrooms, and students mastered the use of *GeoGebra* and achieved the appropriate level of mathematical knowledge in analytic geometry, instead of a written assessment tests, we organized exams using *GeoGebra* available, on the computers in the classroom with the Internet access. Applets were sent to a teacher by e-mail, and subsequently reviewed and evaluated. In classes with the regular program, for a similar methodological units, students were also able, if they wanted, to use *GeoGebra* in the development of written exercises and written assignments, anticipated by work plan. The response of students to this way of teaching and learning was in a very high percentage (90% of students decided to use ICT). Achievements have also been high. The resulting score was equal with other estimates obtained by other techniques of assessment (oral answers, class activities, project work, written assignments and other exercises).

Although there is a possibility of using Serbian language in *GeoGebra*, we insisted on the use of English, in order for students, to learn the proper mathematical terminology. If a student is not familiar to a term in English, he or she can switch to Serbian in the language selection menu and use the command in the native language. After being returned to English, one can easily learn a new term, simply hover over the resulting expression. In other mathematical applications and web services that we use, English is the only language used, so it is more appropriate to learn all the terms above.

Using prepared teaching materials, a teacher saves time required for presentation and explanation of the conclusions in the processing threads. For example, to construct a sine function using the accessories needed to draw requires at least two academic hours. Students usually fail to draw at the same pace as the teacher, because of lack of concentration and experience. Using a prepared dynamic *GeoGebra* applet, it is possible to explain the graph of sine functions in much shorter time (not more than 15 minutes). During the same class, it is possible to examine the shape of functions. If students do not draw during the same class, but do it as homework, having already shown applet, the class will be more interesting and successful. An outcome and learning achievement at the time like this is much bigger and more effective. The time that we "saved" in such a way of learning can take advantage of the processing of a number of examples.

A teacher now has the role of a moderator because he or she directs the student's work. This ensures that the student who was passive before, becomes an active participant in the educational process. Changing roles in the teacher-student relation ensures the responsibility of a student to the final outcome of learning increases. The experimental classes and their curriculum, methodically different, enable that the view point shifts from teacher to student, as a central figure in teaching and learning process. In that way, the requirements of the *Guidelines* [4] on the implementation of **hybrid** learning is fulfilled completely.

Example 1. *Applet for construction of the function* $y = \cos(x)$, [24].

In the teaching of mathematics in Serbia, the graph of a cosine function is usually constructed by shifting the graph of the sine function by $\pi/2$ to the left of the origin, something that the students do not fully understand. We made a dynamic model in *GeoGebra* (Figure 1) that is an effective way to show the construction of the cosine function independently of the graph of the sine function. Point M on the trigonometric circle is determined by the angle α. The cosine of the angle is the numeric value of x coordinates of the point M (M_1, the projection of M on the cosine axis). To draw a point B_1 of the cosine function this value needs to be transferred from x to y axis. This was solved by transferring across the circle of the radius equals to the length of the straight line segment OM_1. The trace of the point B_1 dynamically shows the construction of the graph.

On the website http://tube.geogebra.org/material/show/id/92006 one can watch the animation, without the installation of *GeoGebra*.
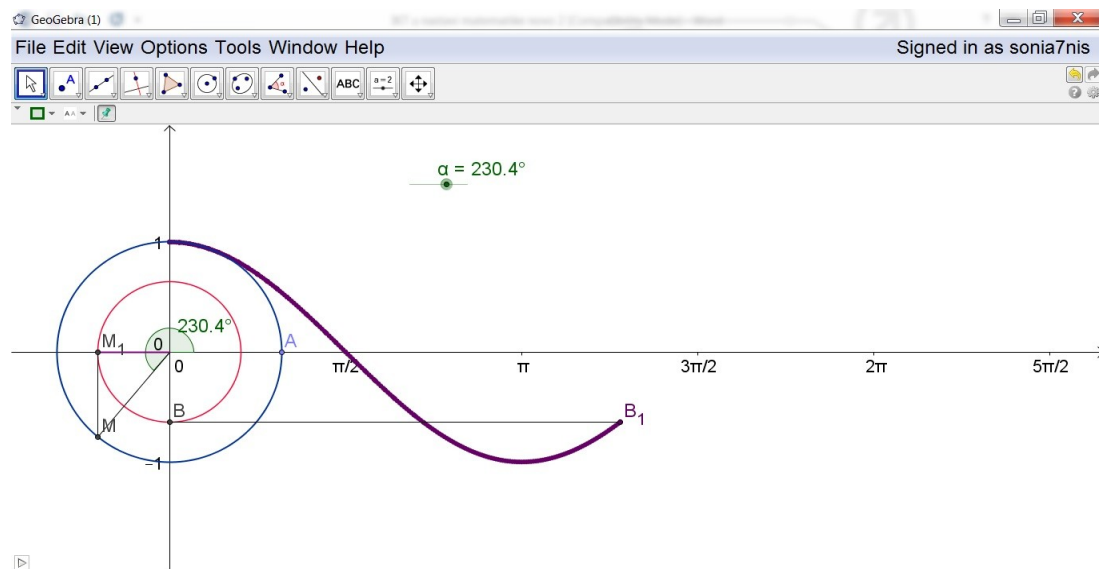

Figure 1. *GeoGebra* applet for construction of the cosine function

Example 2. *Finding roots of complex numbers* [25].

Finding roots of complex numbers is an area that students find very hard to understand. We made an applet in *GeoGebra* that displays the solutions of n-th root, $\sqrt[n]{z}$, of an arbitrary complex number z. Free input is used to enter the real and imaginary components of z. The slider for n determines which root we are looking for, and the slider for k ($0 \le k \le n-$) determines which solution is displayed (Figure 2). We posted the applet on the website http://tube.geogebra.org/material/show/id/303675.
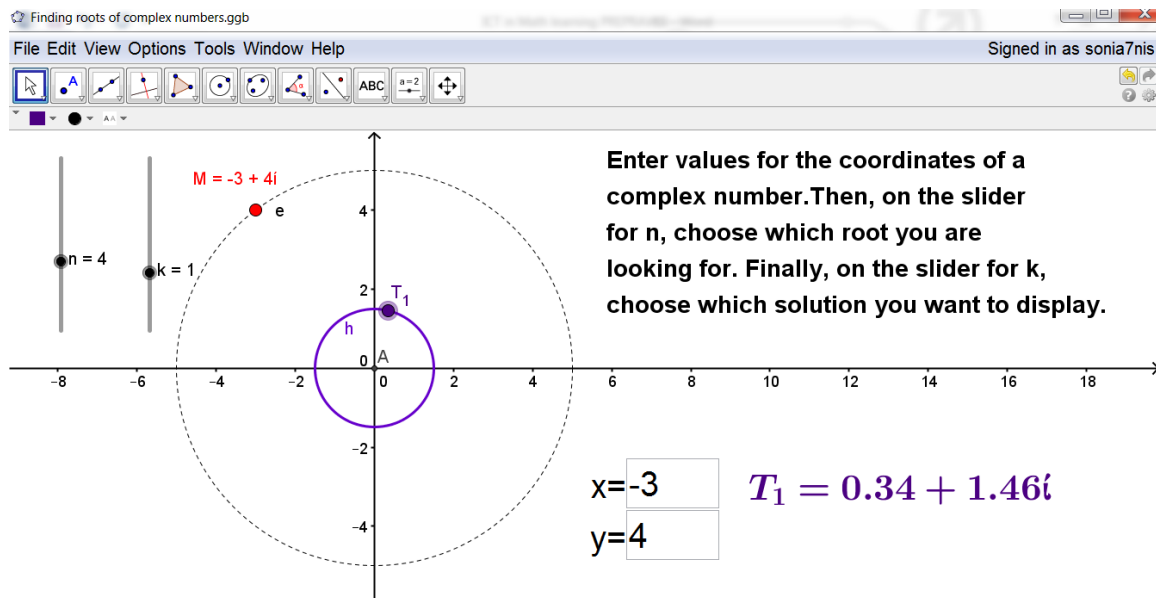
Figure 2. *GeoGebra* applet for finding roots of a complex number

Example 3. *Application in other subjects***:** *Amplitude modulation of signals* [26].

It was noticed that the learning outcomes are the best when different subjects are correlated, so students can observe an application of knowledge from one subject to another one. The authors have applied knowledge about sine and cosine functions introduced during the previous week of instruction in the second grade of *Mathematics*, for understanding the concepts of amplitude modulation of signals in the subject *Fundamentals of telecommunications*. Collaborative work of two teachers in processing the same lesson suits the students very well, and the level of achievement and interest of the students was very high.
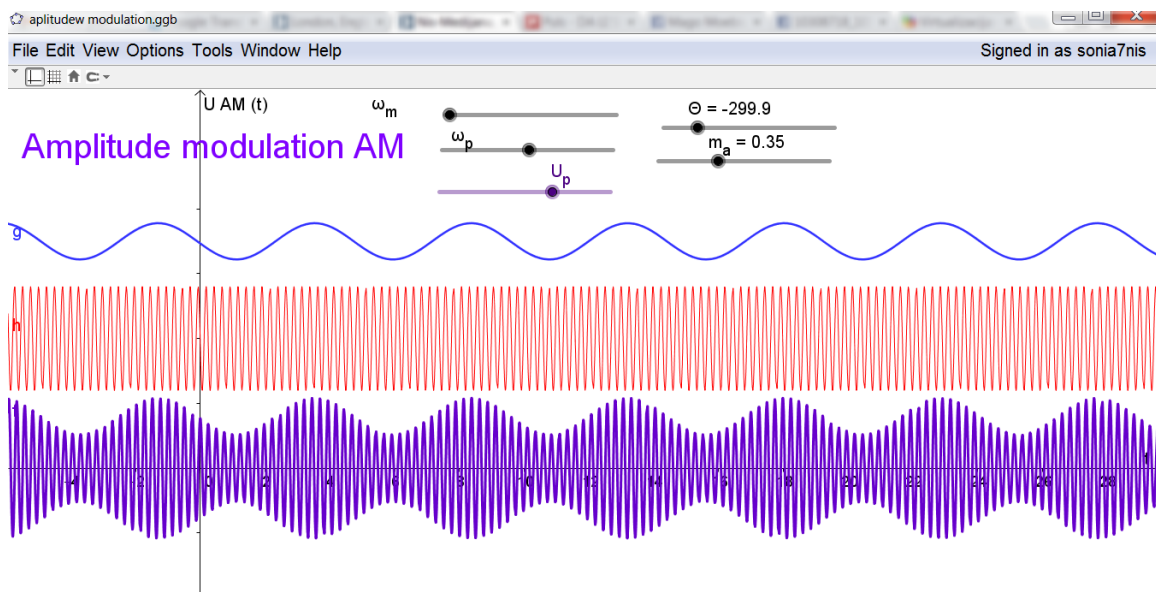
Figure 3. The applet in *Geogebra* that is used in the collaborative lesson

In the applet shown in Figure 3, one can animate all the sliders that show the values that are included in the formula for the modulation of signals. This is a dynamic model of the amplitude modulation of signals. By switching to the Algebra view in *GeoGebra*, students can easily notice the application of the trigonometric functions, since the signals are

constructed by the sine and cosine functions. Benefits are multiplied, because students recognize the correlation of mathematics with technical subjects, and are able to master new knowledge both in mathematics and in telecommunications much easier. The applet is posted on the website http://tube.geogebra.org/material/show/id/303457 . Figure 4 shows a scene during the collaborative lesson.
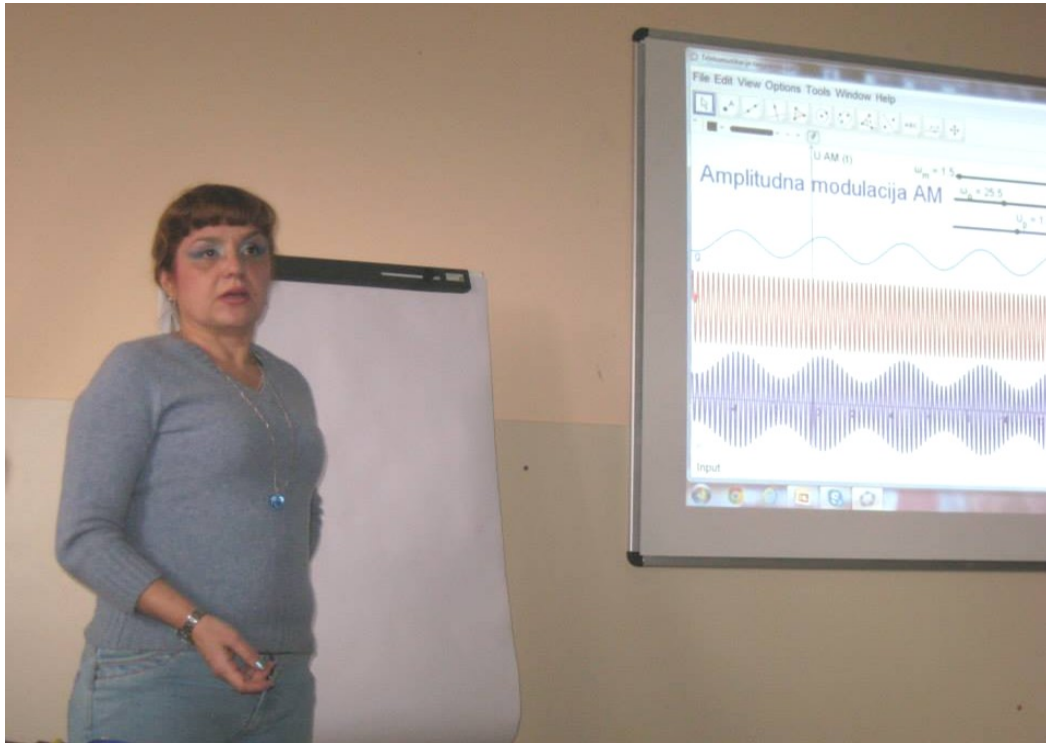

Figure 4. A collaborative lesson

4.2. *Wolfram Alpha* **Internet service**

Due to the high cost of the software package *Wolfram Mathematica*, and in accordance with using only licensed or free software, we turned to the use internet service *Wolfram Alpha*. The use of *Wolfram Alpha* is free. There are Apps that allow the use of the services on smartphones or tablets. In addition to the strictly defined queries, input can be free input in English. Students could use *Wolfram Alpha* to solve not only mathematical problems but also problems in other fields.

Wolfram Alpha offers the possibility of *step-by-step* presentation of solutions. If we use the service in-class (or for homework), we may check our solutions and results that we get by working in the classical way. Also, students can get some other ideas, by examining the set of solutions that the service provides. *Wolfram Alpha* is suitable for use in other, more technical subjects where students need quick usable results.

Example 4. *Derivative of a function* from the *Wolfram Alpha* site, free input.

For a given query, we get a large amount of useful information from *Wolfram Alpha* about the term we were looking for. In this example, we ask for the derivative of a function. From the graph of the derivative function it is easy to see its sign, and therefore it can be used to investigate monotonic behavior of the original function as well as its extreme values.
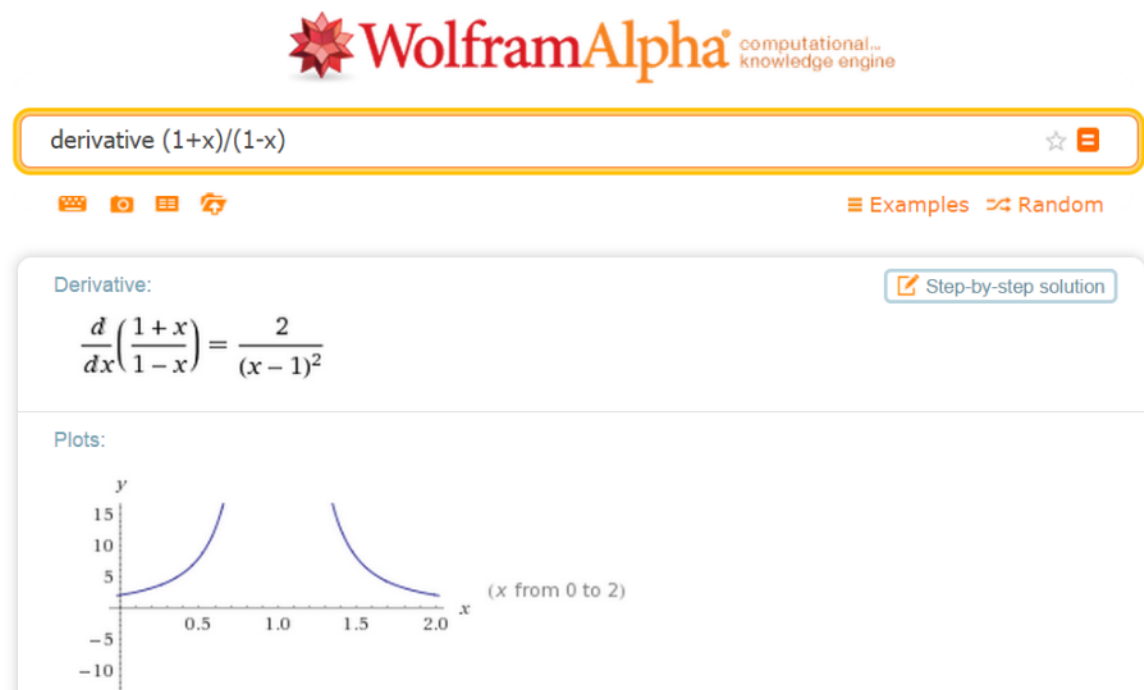
Figure 5. Derivative of a function in *Wolfram Alpha*

Abramovich [27] highlights the appropriate use of technology as support system of knowing and knowledge construction and suggests ways of developing mathematical problems to be suitable for modern teaching, in particular for using *Wolfram Alpha*.

Implementation in practice of *Wolfram Alpha* service can be seen in the paper [28].

Theoretical background of computational experimental approach and discussion about the role of mathematics education reform in using computers in the secondary school education can be found in [29].

The students of the experimental classes of the ETŠ *"Nikola Tesla"* were allowed to use *Wolfram Alpha* during written exams under the controlled conditions (the third and the fourth grade, school year 2010/2011, and 2011/2012). The teacher oversaw the appropriate use of the application (Figure 6). Students were able to check their work and make changes and modifications on time.

4.3. Moodle Learning Management System

Moodle is a LMS (Learning Management System) designed to support both teaching and learning, [30]. It is the framework that handles all aspects of the learning process. The system delivers and manages the instructional content, provides students' independent study, and supports an organized approach to learning. It enables the systematic monitoring of learning outcomes as well as the whole process of learning, both by students and by teachers. Each student can progress at their own pace. It assesses the learning goals for a student or for whole class, tracks the progress towards meeting those goals, and collects and presents data for supervising the learning process as a whole. It also supports the instructional design principles.

Placing *Moodle* platforms on the school website [31] changed the method of conservation of teaching materials from lectures and adding domestic and written assignments. At school, all teachers have their own courses and each student his or her account. A student can see only his or her own account and can only control their own

attachments. Students, by accessing their account for the finished course, can see the lesson, and set attachments to be downloaded to their computer. For example, students can download *GeoGebra* files to their computer and see the analysis of the manner of constructing certain applet.

Students upload homework on *Moodle*. The teacher checks them, evaluates and makes comments. Time for attaching homework is limited as shown on the website. Students are informed in advance of deadlines, both during the F2F class and on the platform. In that way, apart from learning (to achieve a certain outcome) we have an educational role, because the students have to perform their tasks in a given period, in the prescribed (agreed) way. Students are thus prepared for their role in the work process after graduation.

Students can evaluate each other homework. We used options of the same platform, where each student gets to review and evaluate several (usually three) problems of their classmates. If more students are viewing the work, then it is possible to introduce a mark for assessment (obtained on the basis of deviations from assessments of other reviews), so it is possible to obtain two marks for the work (one for the work itself and the other for the assessment of other students work). Both aspects of this evaluation are being used, upbringing and education. We also used the opportunity of collaborative project work (group of students working on a joint problem).
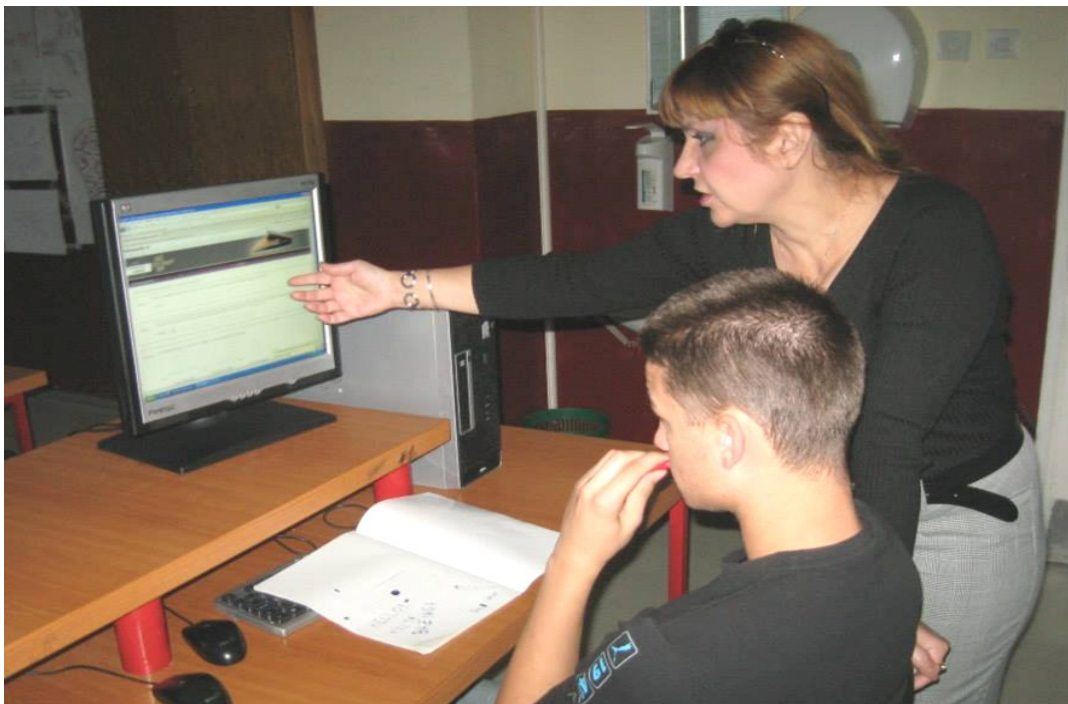

Figure 6. A scene during a lesson

It is also possible to make **Quizzes**, exercises that are done at home, and for the assessment that are done at the school, under controlled conditions (Figure 6). The design of quizzes for mathematics in *Moodle* is not easy, because:

- *Free entry is not suitable for essay answers*. It is not possible to format the answers because editor answer does not exist. Essay responses are manually evaluated, which means that students do not get immediate results of the test, which is not suitable for this type of testing.
- *Choosing the type of questions is a particular problem* (pairing, multiple choice, yes-no, etc.).

- *Questions should be approximately of the same difficulty*.
- *Teacher has to decide which size of the database is the best* (if all the students have the same questions, with the possibility of different order, or not? Then the base must have a larger number of questions, divided by categories of questions).
- *Calibrating tests is almost impossible to do* if only one teacher uses the test. This is possible if he or she is a member of a team that makes the test (including the pedagogue / psychologist). If the teacher uses the same test over several years, it is possible to calibrate it.
- *Penalty points, yes or no*? Penalty points at the age of 15-19 years old are not recommended on the basis of pedagogical recommendations. However, if all questions are YES or NO, a teacher should use penalties. By a proper choice of the variety of issues, using penalties can be avoided.

We used testing, both for the purpose of exercise and assessment. This assessment is equally valued as estimates obtained by other techniques of assessment and its impact on the final mark is not excessive. Although it seems that such testing would be appropriate to students, they should first become accustomed to the use of *Moodle*. Therefore, such testing should be avoided at the beginning of the first grade. Tests were like classical written tasks (five questions, all responses carry the same number of points). For the YES or NO answers, we used penalties, -10%. Students were informed about the existence of penalties and for what kind of issues. Estimated time was half an hour. The students received feedback immediately, and were able to review their solutions from home (Figure 2).

Example 5. *Test for quadratic function*, in duration of 30 minutes.

Students were given five questions. Only one of them is required computation that the students performed on a given sheet of paper. Students had to enroll the answer to this question in a box. Figure 7 is an example of selecting parameters of the quadratic function $y=ax^2+bx+c$.
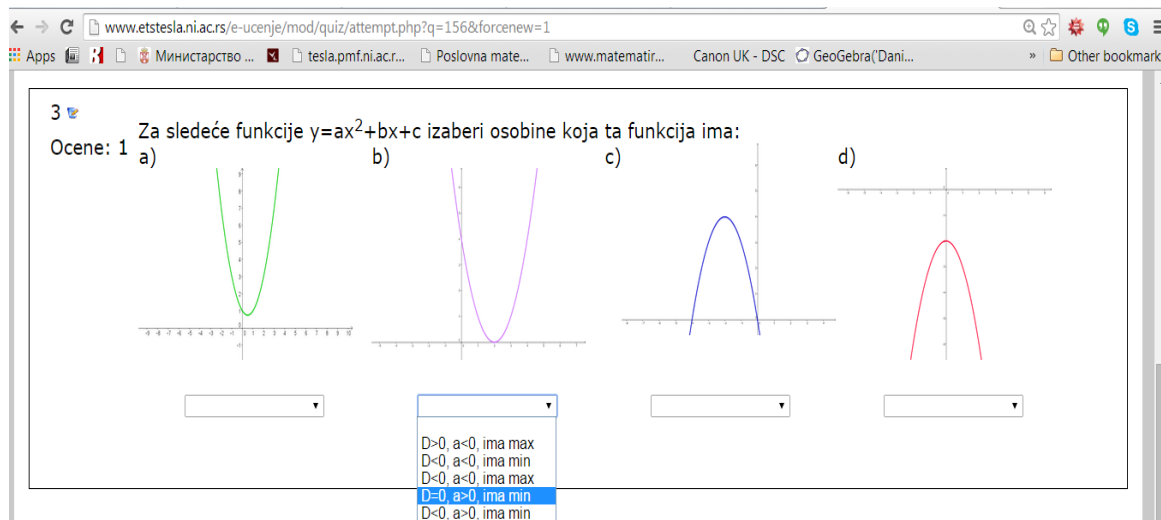

Figure 7. A Quiz in *Moodle*

The curriculum is constantly available to students via computer, and even on mobile phones. If a student is absent from school, he or she can find all the processed lessons and homework posted on the *Moodle* platform [32]. The school enabled students who did not have a computer at home or did not have Internet available, to use computers in the library throughout the day. Students do it willingly. Thus social aspect is also completely fulfilled.

Parents have their own account, and thus may have access to material processing and mandatory student work.

Each student can progress at their own pace and the teacher can monitor the activities and progress for each student. *Moodle* has a large possibility of statistical data processing, according to various criteria (for tests, activities, assignments, lessons, etc.) [33].

By using *Moodle* we are approaching on-line learning. Hybrid model is present completely. This is the way we do in everyday teaching, for all classes, both regular and experimental.

5. CONCLUSION

The authors concluded that the learning process of mathematics in ETS *"Nikola Tesla"* in Niš, Serbia, is organized in the most modern way in technological and methodological terms, with respect to all pedagogical norms. An important aspect of this approach is the enhancement of students' achievements. Students gain more durable and consistent knowledge, which allow them to make progress in the field of mathematics and be successful in their further integration into the mainstream mathematical education at colleges and universities in Serbia and elsewhere.

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